Average Match Quality over the Business Cycle[*]

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Abstract

This paper examines how the business cycle impacts the average quality of an employer-employee match. I study a model of the labor market with on-the-job search, aggregate uncertainty, and heterogeneous match qualities. I test two theories: the *cleansing effect*, whereby the low quality matches are destroyed during recessions, and the *sullying effect*, whereby firms post fewer vacancies during recessions and workers have fewer opportunities to move up the job ladder. I find that the sullying effect dominates and that average match quality is procyclical due to increased hiring out of unemployment during recessions. I extend the model to allow for an exogenous minimum wage and show that neglecting to account for the cyclicality of match qualities can lead to miscalculating the effects of the policy.

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Introduction

Is the average quality of an employer-employee match procyclical? How does the business cycle affect the distribution of match qualities in the economy? Does this vary across worker characteristics such as educational attainment? If so, what are the welfare implications for the evaluation and implementation of labor market policies?

To answer these questions, I study a model of the labor market with several key features which allow me to capture the effects of recessions on average match quality, as well as study the reallocation of workers and firms over the business cycle. First, workers and firms form matches which differ by their quality, assumed to be time invariant. Second, on-the-job search ensures that workers are constantly searching for better opportunities. This induces a job ladder in match quality, i.e. workers endogenously sort into better matches over time as they receive poaching offers from competing firms. Job duration is thus increasing in match quality, with workers unwilling to accept poaching offers from firms which would yield a lower match quality. Third, aggregate uncertainty is governed by a productivity shock which affects the per period output of a given match. Lastly, I allow for endogenous separations so that only matches which are jointly profitable to the worker and to the firm are created and retained. This decision crucially depends on the joint value of a match, which is a function of the match quality and the aggregate state of the economy today, as well as what is expected to happen to the worker and the firm next period, and into the future.

Next, I partition the labor market by workers' educational attainment and separately estimate the model for each group. The model parameters are jointly estimated using the Simulated Method of Moments with data from the Current Population Survey during the period Dec2001-Dec2019.

I use the estimated model to analyze the cyclicality of the average match quality in the economy. Two forces in the model impact its cyclicality. First, the Schumpeterian *cleansing effect*[1], the idea that recessions consist in a destruction of the worst matches and a sorting of workers to better matches, translates to an increase in the average match quality. Second, the *sullying effect* coined by Barlevy (2002), the idea that recessions impede reallocation because firms post fewer vacancies and workers cannot sort into better jobs as easily, implies a decrease in the average match quality. I find that average match quality, as measured by its correlation with unemployment, is strongly procyclical across all education groups.

To understand why the sullying effect dominates and how both effects are present in the model, I simulate the transition path back to the steady state equilibrium following an unexpected one-period negative shock to productivity. At impact of the negative shock, disproportionately more low matches are separated than high matches because of endogenous separations. Thus, the average match quality initially increases. However, as the number of vacancies posted by firms recovers, so does hiring. I find that hiring out of unemployment dominates hiring out of employment and since, on average, matches hired out of unemployment are of

[1] Schumpeter viewed recessions as a time where resources get efficiently reallocated to more productive uses. In Schumpeter (1942), he describes this as a process "that incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism."

lower quality due to selection, the average match quality decreases.

I then explore how the strengths of the sullying and cleansing effects depend on the depth of the recession. To do so, I look at periods where the productivity level decreases, what I refer to as a recession in the context of the model, and, in addition, condition on the level of aggregate productivity. This is to capture the possibility that a decrease in aggregate conditions from the best to second-best state is inherently different than a decrease from the second-worst to the worst state. I find that in periods where the latter holds, both the cleansing and sullying effects are present and it is not clear which dominates. However, as the severity of the recession decreases, the cleansing effect becomes less prominent. This relationship holds regardless of education.

To further understand the mechanisms impacting the cyclicality of the average match quality, I study its interaction with the level of mismatch in the economy. Since the model is written in terms of match quality, there is a clear notion of mismatch: all workers who are not in the highest quality matches, and thus producing less than they would in the best matches, are deemed to be mismatched. I introduce three measures of mismatch and analyze how they depend on the business cycle and interact with average match quality. All three measures are strongly countercyclical and decreasing in educational attainment.

Finally, I extend the model to allow for an exogenous minimum wage policy. I study its effects on both the stationary and stochastic versions of the model. In the stationary model, there is a large employment effect: total employment strongly decreases in the presence of a minimum wage. This decrease is especially large for the lower educated groups which are most impacted by the policy. I study how employment and the percent of the employed who receive the minimum wage vary across the cycle and by education. I then study the introduction of the minimum wage policy in the full stochastic model, which, in contrast to the stationary version, has time varying average match quality. I explore its relationship with the effects of the minimum wage policy. Neglecting to account for a procyclical average match quality can lead to miscalculating the effects of the policy.

Related Literature. This paper is not the first to think about the cyclicality of match quality. Several empirical papers, dating back to Bowlus (1995), and more recently Foster, Grim, and Haltiwanger (2016), Mustre-del Rio (2019), and Baydur and Mukoyama (2020), proxy the quality of a match by its duration and find that it is procyclical. However, as noted by Mustre-del Rio (2019), measuring match quality as job duration has important limitations which lead to biased estimates of its cyclicality. Other studies, such as the influential works of Kahn (2010) and Oreopoulos, von Wachter, and Heisz (2012), use the wage as a proxy for match quality and find that labor market earnings are lower for workers who join the labor force during recessions. This paper contributes to this strand of the literature by providing a structural estimate of the cyclicality of the average match quality without needing to rely on imperfect measures of the quality of a match.

This paper also relates to a branch of the literature studying mismatch in the labor market. Of those, this paper is closest to Baley, Figueiredo, and Ulbricht (2021) who also study a framework that allows for the coexistence of both the cleansing and sullying effects. Their focus is on average mismatch, which they find to be procyclical thus implying that the cleansing effect dominates.

Furthermore, this paper contributes to a large literature interested in reallocation over the business cycle which studies models of the labor market with both aggregate uncertainty and two-sided heterogeneity (see, e.g., amongst many others, Burdett and Mortensen, 1998; Barlevy, 2002; Postel-Vinay and Robin, 2002; Menzio and Shi, 2011; Moscarini and Postel-Vinay, 2013; Lise and Robin, 2017; and Schaal, 2017). Of these, the model is closest to that of Barlevy (2002) with a couple distinctions which I discuss in the previous section.

Finally, this paper relates to a growing literature studying the effects of a binding minimum wage policy. It is closest to Flinn (2006) with the main difference being that I not only study the effects of a minimum wage in a stationary environment, but also in the case where there is aggregate uncertainty.

Outline. The rest of the paper is organized as follows: Section 1 presents the model environment. Section 2 describes the data and estimation procedure Section 3 explores the the effects of the business cycle on the average match quality. Section 4 extends the model to allow for an exogenous minimum wage and analyzes its effects in both the stationary and stochastic versions of the model. Section 5 concludes.

1 Model

In this section, I present a model of the labor market with heterogeneous match quality, aggregate uncertainty, and on-the-job search. The model is closest to the one developed by Barlevy (2002) with a few distinctions. First, the only heterogeneity present in the model is through the quality of a match. Barlevy (2002) has two-sided heterogeneity and shows that under a particular set of assumptions[2] the model can be written in terms of match quality. I directly write the model in terms of match quality since that is the object of interest in this paper and it allows me to avoid making a similar set of assumptions. Second, I maintain Nash bargaining throughout whereas he assumes wages take the form of a piece-rate on output. This ensures that all decisions are jointly efficient between the worker and the firm. Imposing an exogenous wage would result in inefficient separations and bias the cyclicality of the average match quality. Third, I assume the quality of a match follows a Zipf distribution, the discrete analog of a Pareto distribution with a finite support. This is a generalisation of Barlevy (2002) who has a uniform distribution and makes it so there is less bunching at the top of the distribution of workers across match qualities. Finally, I allow the value of home production to vary across states to allow for the possibility that the opportunity cost of employment is different in recessions and in booms which also impacts the cyclicality of the average match quality.

[2]He assumes that worker and firm types take on the same number of values and worker types are uniformly distributed. Furthermore, he makes a functional form assumption on the mapping from worker and firm types to the quality of a match, similar to the one in Marimon and Zilibotti (1999). All these assumptions make the value functions independent of the distributions of workers and firms across types. Finally, he only considers symmetric equilibria where firms post the same number of vacancies.

1.1 Model environment

Environment. Time is discrete and indexed by $t \in \{1, 2, 3, \dots\}$. The economy is populated by a unit mass of workers and firms. The state of the economy in t is given by (s_t, e_t) where s_t is aggregate productivity with transition probability $\pi(s_{t+1}|s_t)$ and $e_t : \{1, 2, \dots, N\} \to [0, 1]$ is a function with $e_t(n)$ denoting the measure of workers in matches of quality n.

Matching. Let v_t denote the number of vacancies posted and u_t the measure of unemployed workers in period t. Note that,

$$u_t + \sum_{n=1}^{N} e_t(n) = 1, \; \forall t. \tag{1}$$

Search is random and I assume all workers search. Hence, the market tightness in the economy is the number of vacancies posted by firms, $\theta_t = v_t/1 = v_t$. The number of new matches in period t is m_t and given by a meeting function $m_t = m(1, v_t)$, assumed to be increasing in both arguments and constant returns to scale. The probability for a worker, irrespective of employment status, of meeting a firm is $p(\theta_t) = m(1, v_t)/1 = m(1, \theta_t)$, where $p(\cdot)$ is strictly increasing, strictly concave, $p(0) = 0$, and $p(1) = 1$. The probability for a firm of meeting a worker is $q(\theta_t) = m(1, v_t)/v_t = m(1/\theta_t, 1) = p(\theta_t)/\theta_t$, where $q(\cdot)$ is strictly decreasing, $q(0) = 1$, and $q(1) = 0$.

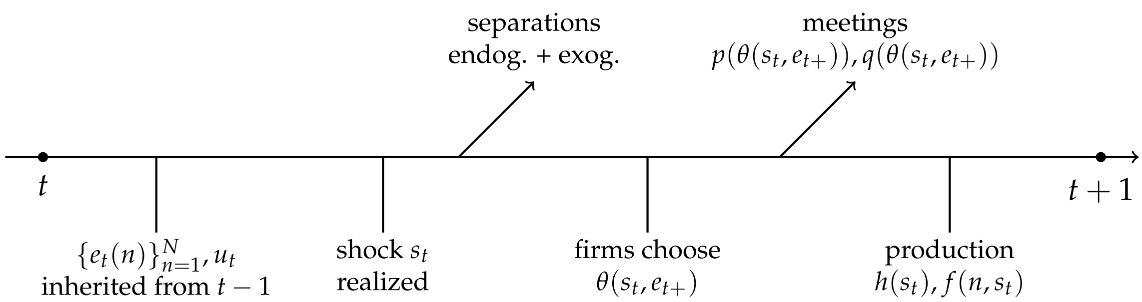
Match Quality. Upon meeting, a worker and firm pair draws a match quality according to a Zipf distribution with shape parameter $\eta \geq 0$ and support $\{1, 2, 3, \dots, N\}$. Hence, the worst matches are of quality 1 and the best have quality N. The probability that a worker and firm pair form a match quality of n is $1/\left(n^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}\right)$ and is independent of the aggregate state of the economy. When $\eta = 0$, this reduces to a uniform distribution. As η increases, the probability of drawing a high match quality decreases. The match is consummated if and only if it is jointly profitable to do so, i.e. if the total match surplus is positive. Note here that there is no notion of learning about the quality of the match over time. As soon as the worker and the firm meet, the quality is observable to both and it is time invariant. The only way for a worker to be in a match of higher quality is to switch firms. Finally, although the distribution from which workers and firms draw their match quality is stationary, the contact rates are not. Indeed, they depend on the endogenous distribution of workers across match qualities and vary across the cycle. So the total probability to the worker, for example, of forming a match of quality n, given by the product $\frac{1}{n^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} p(\theta)$, will be time varying.

In the model, the only heterogeneity present is at the level of the match. To address this restriction, in the quantitative section I partition the labor market by educational attainment and estimate the model separately for each group. The assumption is then that there are no flows between these different groups. I discuss this in more detail in Section 2.4.

1.2 Timing

The timing in the model within a period is illustrated in Figure 1. At the beginning of period t, the distribution of workers across match qualities is inherited from the previous period $t - 1$.

Figure 1: Within period timing



Then the productivity shock s_t is realized. Next, matches are both exogenously separated at rate δ and endogenously separated if the joint value of the match $S(n, s_t, e_t)$ is negative. Firms then observe the distribution of workers who survive separations across match qualities, given by

$$\begin{aligned} e_{t+}(n) &= \begin{cases} (1-\delta)e_t(n) & \text{if } S(n, s_t, e_t) \geq 0 \\ 0 & \text{if } S(n, s_t, e_t) < 0 \end{cases} \\ &= (1-\delta)1\{S(n, s_t, e_t) \geq 0\}e_t(n), \ \forall n \end{aligned}$$

as well as the productivity shock and decide how many vacancies $\theta(s_t, e_{t+})$ to post. Then meetings occur at rates $p(\theta(s_t, e_{t+})), q(\theta(s_t, e_{t+}))$ and the worker-firm pair draws a match quality. Finally, production occurs with a match of quality n producing $f(n, s_t)$ units of output and an unemployed worker producing $h(s_t)$. $f(\cdot)$ is assumed to be strictly increasing in n. Finally, period $t+1$ begins with $\{e_{t+1}(n)\}_{n=1}^N$ and u_{t+1}.

As opposed to the Diamond-Mortensen-Pissarides paradigm standard in the literature and almost all search and matching models of the labor market, I allow home production to depend on the state of the economy. This decision is first motivated by the fact that the value of $f(n, s)$ relative to $h(s)$ will be crucial in determining which matches remain feasible during recessions and so will have a direct impact on the cyclicality of the average match quality. Second, this choice is aligned with Chodorow-Reich and Karabarbounis (2016) who provide convincing evidence that the flow opportunity cost is strongly procyclical and hence not constant over time.

1.3 Laws of motion

As in Lise and Robin (2017), I make the distinction between the stocks of employed and unemployed workers right after the realization of s_t and the stocks at the end of the period, after matching between workers and firms has occurred. Denote by u_{t+} the number of workers who are unemployed following separations in period t. If a match is no longer feasible in period t after the realization of s_t, it separates. Otherwise, it is exogenously destroyed at rate δ. Therefore, the probability that a match of quality n separates in period t is

$1\{S(n,s_t,e_t) < 0\} + \delta 1\{S(n,s_t,e_t) \geq 0\}$. u_{t+} thus evolves according to

$$u_{t+} = u_t + \sum_{n=1}^{N} e_t(n) \Big[1\{S(n,s_t,e_t) < 0\} + \delta 1\{S(n,s_t,e_t) \geq 0\} \Big] \tag{2}$$

where the second term captures the number of workers flowing into unemployment from employment. The number of unemployed workers in $t+1$ is the number of unemployed workers after separations who do not match with a firm. Therefore, the number of unemployed workers in the economy follows the law of motion:

$$u_{t+1} = u_{t+} \Bigg[1 - \underbrace{p(\theta(s_t,e_{t+})) \sum_{n=1}^{N} \frac{1}{n^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} 1\{S(n,s_t,e_t) \geq 0\}}_{\text{prob. receiving \& accepting offer}} \Bigg]. \tag{3}$$

Finally, the law of motion for the stock of employed workers in matches of quality n is

$$e_{t+1}(n) = \underbrace{\left(1 - p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \right)}_{\text{prob. no poaching}} e_{t+}(n) + \frac{1}{n^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} p(\theta(s_t,e_{t+})) \left(1 - \sum_{k=n}^{N} e_{t+}(k) \right). \tag{4}$$

The first term corresponds to workers inherited from the previous period, i.e. who do not separate into unemployment or move up the job ladder. Once a worker survives separations, they meet with a firm at rate $p(\theta(s,e_+))$ and only accepts an offer that grants them a higher value than the continuation value of their current match. I assume that if a worker draws the same quality n then they remain at their current firm.[3] In Proposition 1, I show the model has a job ladder in match quality and so this will only happen if they draw a match quality strictly higher than their current one, i.e. from the set $\{n+1,\ldots,N\}$. The second term corresponds to the inflow from other matches and unemployment. Because of the job ladder, only workers in lower quality matches would accept an offer. All unemployed workers who contact a firm move into employment if the worker surplus of the match is positive. So the number of workers who would accept an offer of quality n is $u_{t+} + \sum_{k=1}^{n-1} e_{t+}(k) = 1 - \sum_{k=n}^{N} e_{t+}(k)$, which is increasing in n.

1.4 Value functions

Worker Value Functions. An unemployed worker produces $h(s)$ units of output in the current period. The next period, if they do not meet with a firm, which occurs with probability $1 - p(\theta(s,e_+))$, they receive the continuation value of being unemployed. If they meet with a firm, the match is of quality n with probability $1/\left(n^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}\right)$ and is formed if and only if it is jointly

[3] This assumption can be micro-founded by introducing a switching cost. Moreover, it avoids the issue highlighted by Shimer (2006) on models with on-the-job search and Nash bargaining.

optimal. The value to a worker of being unemployed is then

$$
\begin{aligned}
U(s,e) = h(s) + \beta \mathbf{E}_{s'|s}\Big[&\big(1 - p(\theta(s,e_+))\big)U(s',e') \\
&+ p(\theta(s,e_+)) \sum_{n=1}^{N} \frac{1}{n^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}} \max\{W(n,s',e'),U(s',e')\}\Big]
\end{aligned}
$$

where the expectation is taken over the productivity shock and $W(n,s',e')$ is the value to the worker of being employed in a match of quality n next period. Using the identity $\max\{a,b\} = \max\{a-b,0\}+b$, this can be rewritten more concisely as

$$
U(s,e) = h(s) + \beta \mathbf{E}_{s'|s}\left[U(s',e') + p(\theta(s,e_+)) \textstyle\sum_{n=1}^{N} \frac{1}{n^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}} \max\{W(n,s',e') - U(s',e'),0\}\right]. \quad (5)
$$

Here it is clear that next period, the worker receives the continuation value of unemployment and, in addition, extracts the surplus from moving into employment if they receive and accept an offer.

An employed worker is paid a wage $w(n,s,e)$. The following period, they separate into unemployment with probability $1-(1-\delta)1\{S(n,s',e') \geq 0\}$[4]. If no separation occurs, they either do not meet with a firm, in which case they receive the continuation value $W(n,s',e')$, or they meet with a firm and the match is formed if it is jointly optimal. I assume that if a worker I show later that this occurs only when the worker meets with a firm and draws a strictly greater match quality. The value to a worker of being employed in a match of quality n is thus

$$
\begin{aligned}
W(n,s,e) = w(n,s,e) + \beta \mathbf{E}_{s'|s}\Bigg[&\underbrace{\Big(1-(1-\delta)1\{S(n,s',e') \geq 0\}\Big)}_{\text{prob. of a separation}} U(s',e') \\
&+ \underbrace{(1-\delta)1\{S(n,s',e') \geq 0\}}_{\text{prob. of no separation}} \Big[\big(1-p(\theta(s,e_+))\big)W(n,s',e') \\
&+ p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}} \max\{W(j,s',e'),W(n,s',e')\}\Big]\Bigg].
\end{aligned}
$$

As with the value of unemployment, this expression can be rewritten as

$$
\begin{aligned}
W(n,s,e) = w(n,s,e) + \beta \mathbf{E}_{s'|s}\Bigg[&U(s',e') + (1-\delta)1\{S(n,s',e') \geq 0\}\Big(W(n,s',e') - U(s',e')\Big) \\
&+ (1-\delta)1\{S(n,s',e') \geq 0\} p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}} \max\{W(j,s',e') - W(n,s',e'),0\}\Bigg].
\end{aligned}
\quad (6)
$$

[4]This is the same as the expression in Equation (2). To see this, note that the probability that the match survives is

$$
\begin{aligned}
1 - \big(1\{S(n,s,e) < 0\} + \delta 1\{S(n,s,e) \geq 0\}\big) &= \big(1 - 1\{S(n,s,e) < 0\}\big) - \delta 1\{S(n,s,e) \geq 0\} \\
&= 1\{S(n,s,e) \geq 0\} - \delta 1\{S(n,s,e) \geq 0\} \\
&= (1-\delta)1\{S(n,s,e) \geq 0\}
\end{aligned}
$$

Hence, the probability that the match separates is $1-(1-\delta)1\{S(n,s,e) \geq 0\}$.

This is an intuitive extension of Equation (5). An employed worker is paid a wage $w(n,s,e)$ and next period is guaranteed the value of unemployment plus the continuation value of the worker surplus of their current match if there is no separation. In addition, they collects the gains, relative to their current match, from potentially moving up the job ladder next period.

Firm Value Function. Finally, the value to a firm of a match of quality n is the output it produces less the wage bill this period plus the expected continuation value of the match next period, conditional on the match not getting separated and the worker not getting poached. After separations have occurred, the worker makes an employment-to-employment transition if and only if they meet with a firm and draws a match quality greater than n, which happens with probability $p(\theta(s,e_+)) \sum_{k=n+1}^{N} \frac{1}{k^\eta} / \sum_{k=1}^{N} \frac{1}{k^\eta}$. The firm value function is thus

$$\begin{aligned} J(n,s,e) &= f(n,s) - w(n,s,e) \\ &\quad + \beta \mathbf{E}_{s'|s}\left[(1-\delta)1\{S(n,s',e') \geq 0\}\left(1 - p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}}\right) J(n,s',e')\right]. \end{aligned} \tag{7}$$

Vacancy creation. The value to a firm of a vacant match is equal to the cost κ of posting the vacancy in the current period plus the expected profits of the vacancy. If the firm does not meet with a worker, it receives the continuation value of the vacant match. The vacancy can be filled by unemployed and employed workers. If the firm meets with an employed worker, the match is formed if and only if the quality created is greater than the one the worker had with their previous employer. As such, for a given match quality n, the number of workers in the economy who would accept to form the match is $u_{t+} + \sum_{l=1}^{n-1} e_{t+}(l)$. Hence, the value of a vacant match to the firm is

$$\begin{aligned} V(s,e) &= -\kappa + \big(1 - q(\theta(s,e_+))\big)V(s,e) \\ &\quad + q(\theta(s,e_+)) \sum_{n=1}^{N} \frac{1}{n^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \left(u_{t+} + \sum_{l=1}^{n-1} e_{t+}(l)\right) \max\{J(n,s,e), V(s,e)\}. \end{aligned}$$

Imposing free entry in equilibrium, i.e. $V(s,e) = 0 \; \forall (s,e)$, yields the free entry condition

$$\kappa = q(\theta(s,e_+)) \sum_{n=1}^{N} \frac{1}{n^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \left(u_{t+} + \sum_{l=1}^{n-1} e_{t+}(l)\right) \max\{J(n,s,e), 0\} \tag{8}$$

which equalizes the cost of posting a vacancy to its expected benefits. This equation pins down the number of vacancies firms decide to post every period, which is equal to the market tightness since there is unit mass of workers searching every period.

Match Surplus. Let $S(n,s,e) := W(n,s,e) + J(n,s,e) - U(s,e)$ denote the present value of a match of quality n. In Appendix A.1, I show that this can be written as

$$
\begin{aligned}
S(n,s,e) = f(n,s) - h(s) + \beta \mathbf{E}_{s'|s} \Bigg[& (1-\delta) \max\{S(n,s',e'),0\} \left(1 - p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \right) \\
& + \alpha(1-\delta) 1\{S(n,s',e') \geq 0\} p(\theta(s,e_+)) \sum_{j=n+1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} S(j,s',e') \\
& - \alpha p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'),0\} \Bigg].
\end{aligned} \tag{9}
$$

The first term in the expectation corresponds to the continuation value of the match conditional on survival, i.e. no separation occurs (either exogenously or endogenously) and the worker does not get poached to a better match. The second term corresponds to the expected surplus the worker extracts from moving up the job ladder. The last term is the foregone surplus the worker loses by being employed. Finally, notice that all future worker and firm decisions appear in the surplus equation.

Proposition 1. *[Job ladder] For a given state (s,e), $S(n,s,e)$ is strictly increasing in n.*

The formal proof of this result is relegated to Appendix A.3. Intuitively, the surplus of a match is simply the expected future stream of output less home production and since output is strictly increasing in the quality of the match, the surplus will be as well.

Wage Determination. Workers and firms split the surplus according to the Nash bargaining solution:

$$
w(n,s,e) = \arg\max_{w} S^W(n,s,e)^\alpha J(n,s,e)^{1-\alpha} \tag{10}
$$

where $S^W(n,s,e) := W(n,s,e) - U(s,e)$ is the worker surplus and $\alpha \in [0,1]$ is the worker's bargaining power. Note that the firm surplus is simply $J(n,s,e)$ because of free entry. This surplus splitting rule yields the standard condition that worker surplus is proportional to firm surplus (see Appendix A.2)

$$
J(n,s,e) = \frac{1-\alpha}{\alpha} S^W(n,s,e) \tag{11}
$$

which implies $S^W(n,s,e) = \alpha S(n,s,e)$ and $J(n,s,e) = (1-\alpha)S(n,s,e)$. So the worker and firm surpluses are also strictly increasing in match quality n. In Appendix A.2, I show that the wage can be expressed as the weighted average of output and home production plus the expected gains from on-the-job search next period.

1.5 Equilibrium

Definition 1.1 (Equilibrium). *A recursive stochastic equilibrium, given a sequence of shocks $\{s_t\}_{t=0}^{\infty}$ and an initial distribution $\{e_0(n)\}_{n=1}^{N}$, is a path for market tightness $\{\theta_t\}_{t=0}^{\infty}$, a distribution of workers across match qualities $\{\{e_t(n)\}_{n=1}^{N}\}_{t=1}^{\infty}$, and a surplus function $S(n,s_t,e_t)$ such that*

1. *θ_t satisfies the free entry condition (Equation (8)) for all t*

2. *$S(n, s_t, e_t)$ solves its recursive formulation Equation (9)*

3. *$\{\{e_{t+1}(n)\}_{n=1}^{N}, u_{t+1}\}_{t=0}^{\infty}$ follow their laws of motion Equation (4) and Equation (3)*

The equilibrium is fully characterized by Equations (1), (4), (8), (9) and (11). Solving for the equilibrium outcomes is not straightforward because of three features of the model. First, the joint surplus of a match depends on its expected future value, i.e. on all possible decisions of the worker and the firm. Second, the joint surplus depends on the cycle in part through the market tightness which is itself a function of the surplus. Lastly, the surplus of a match of quality n depends on the future surpluses of all other match qualities $-n$.

I use the following solution method to circumvent these issues. I approximate the surplus as a polynomial in the distribution $\{e_t(n)\}_{n=1}^{N}$ using the collocation method. Then, in a given period with the distribution $\{e_t(n)\}_{n=1}^{N}$ inherited from the previous period, Equation (8) is used to invert the market tightness θ_t. Finally, I iterate forward using Equation (4) to get the new distribution $\{e_{t+1}(n)\}_{n=1}^{N}$. The key step is approximating the surplus equation for each triple (n, s, e). The full description of the solution method, as well as the computational and implementation details, are relegated to Appendix B.1.

2 Estimation and Data

In this section, I describe how the model is estimated. I first complete the description of the model with its parametric specification and then explain the procedure used to estimate the model parameters. Next, I describe the data and the set of moments used for estimation. Finally, I present the parameter estimates and assess the model fit.

2.1 Parametric Assumptions

State Space. I allow for five different states and specify the magnitude of the productivity shocks with the parameter step as follows:

$$z(s_1) = 1 - 4\text{step}, \;\; z(s_2) = 1 - 3\text{step}, \;\; z(s_3) = 1 - 2\text{step}, \;\; z(s_4) = 1 - \text{step}, \;\; z(s_5) = 1.$$

For example, if step $= 0.025$ then the worst state corresponds to a 10% productivity shock relative to the highest state. I assume the transition matrix is parameterized by $\{p_1, p_2\}$ and takes the form

$$\Pi = \begin{bmatrix} 1 - p_1 & p_1 & 0 & 0 & 0 \\ p_2 & 1 - p_1 - p_2 & p_1 & 0 & 0 \\ 0 & p_2 & 1 - p_1 - p_2 & p_1 & 0 \\ 0 & 0 & p_2 & 1 - p_1 - p_2 & p_1 \\ 0 & 0 & 0 & p_2 & 1 - p_2 \end{bmatrix}$$

where $\Pi(i, j)$ is the probability of moving from state i to state j.

This specification for Π involves two main restrictions. First, it doesn't allow the state to increase or decrease by more than 1 from one period to the next. In practice, I simulate the model at a monthly frequency which makes this assumption less restrictive since month-to-month jumps or falls of more than one state are rare in the data. Second, it implies, for example, that the probability of going from the worst to the second-worst state is the same as going from the second-best to the best state, i.e. $\Pi(2,1) = \Pi(5,4) = p_2$. This is more restrictive since one could imagine allowing the rates at which the economy recovers and worsens to differ. In a previous version, I relax this assumption, but find that it does not improve the quality of the model fit nor does it impact the cyclicality of the average match quality[5].

Production. Next, the output of a match of quality n in state s is given by the production function

$$f(n,s) = z(s) \times (\phi_1 + \phi_2 n + \phi_3 n^2)$$

where $\{\phi_1, \phi_2, \phi_3\}$ are parameters. I assume home production is proportional to the state of the economy:

$$h(s) = z(s) \times \gamma$$

Matching Function. Finally, the last element of the model is the matching function which specifies the number of new meetings as a function of aggregate search and vacancies. I follow den Haan, Ramey, and Watson (2000) and assume it takes the form

$$m = m(1,\theta) = \frac{\theta}{(1+\theta^{\iota})^{1/\iota}}$$

where $\iota > 0$. This functional form ensures that the meeting probabilities $p(\cdot), q(\cdot)$ are bounded between 0 and 1. I turn now to describing how the model parameters are estimated.

2.2 Estimation

Externally set parameters. In addition to the number of states, I externally set the following parameters. The number of different match qualities is set to $N = 10$. This could in principle be arbitrarily big, but I found that 10 was simultaneously large enough to allow for meaningful endogenous sorting and small enough that it doesn't significantly increase the computational cost of the solution method. The monthly discount factor is set to $\beta = 0.996$. Finally, I set the worker's bargaining power to $\alpha = 0.5$. Note that this implies, from Equation (11), that the worker and firm surplus are equal every period, $S^W(n,s,e) = J(n,s,e)$, $\forall n,s,e$. In future work, I plan to estimate α, but do not do so here because none of the moments have a clear one-to-one mapping with bargaining power. Finally, note that the model can be extended to

[5] In the previous version, I also include $\{p_3, p_4\}$ where $p_3(p_4)$ is the probability of moving up (down) by two states. Although adding these two parameters did not significantly increase the computational burden of the estimation routine, it did affect the precision of the model estimates. This is because the moments that roughly pin down $\{p_1, p_2\}$ are the same that would roughly identify $\{p_3, p_4\}$. Moreover, since the model is estimated at a monthly frequency, the estimated p_3, p_4 would turn out to be very small and not statistically different from 0. Hence, they would have little to no impact on the sorting of workers over the business cycle. For these reasons, I set $p_3 = p_4 = 0$.

allow α depend on the match quality[6]. This might be a reasonable extension to consider if one thinks that bargaining power varies across worker or firm characteristics. Of course, it then becomes even harder to estimate bargaining power if it is heterogeneous across match qualities. For the purposes of this paper, I stick to a homogeneous and exogenously set α.

Jointly estimated parameters. I use the Simulated Method of Moments (SMM) to jointly estimate the 11 remaining parameters of the model denoted by $\varphi := \{\delta, \kappa, \phi_1, \phi_2, \phi_3, \text{step}, p_1, p_2, \iota, \gamma, \eta\}$. The SMM estimator is the solution to

$$\hat{\varphi} = \arg\min_{\varphi} \sum_i w_i \left(\frac{m_i - \hat{m}_i(\varphi)}{m_i} \right)^2 \tag{12}$$

where i indexes the moments, m_i is the data moment, $\hat{m}_i(\varphi)$ is the corresponding moment simulated from the model under φ, and w_i is the weight. In practice, I use 15 moments which I describe in Section 2.5. To find the global minimum, I use the Markov Chain Monte Carlo (MCMC) methods introduced by Chernozhukov and Hong (2003) and extended to allow for parallel tempering by Baragatti, Grimaud, and Pommeret (2013) which have become increasingly popular to estimate structural models in economics (see, e.g., amongst many others, Lise and Robin, 2017; Oswald, 2019; Balke and Lamadon, 2020; and Jarosch, 2021).

The main advantage of MCMC, as opposed to standard gradient based approaches, is that it is derivative free. Hence, it only requires evaluating the objective function repeatedly and I do not have to worry about discontinuities. The disadvantage of this method is that it can take a very long time for the chain to converge. In practice, I solve and simulate the model for 3,000 periods (250 years), but some of the parameters are not precisely estimated[7], as I discuss in Section 2.6. See Appendix C.2 for a description of the algorithm and computational details.

2.3 Data description

The model is estimated on monthly U.S. data from December 2001 to December 2019. The data for all but one series used in this study come from the Basic Monthly Current Population Survey (CPS). The CPS is a monthly survey of approximately 60,000 households dating back over 60 years.

Although this survey is primarily designed for cross-sectional analysis, it contains a panel dimension that can be taken advantage of to follow households and individuals over short periods of time. In each period, the sample is split into eight disjoint "rotation groups". Every household is interviewed for four consecutive months then is removed from the sample for eight months. So in any given month, for example, 1/8th of the sample is answering questions during their first of four consecutive months in the sample. This is done so that for each

[6] In fact, in this case the surplus equation is identical to Equation (9), except that all the α's are now indexed by match quality and so do not come out of the sums.

[7] The computational constraint is amplified here since I am estimating the model separately for each education group. One estimation routine, i.e. for one education group, takes approximately 44 hours, hence the full estimation routine takes close to two weeks. To get more precise estimates would require significantly increasing both the number of chains and iterations, which would greatly increase the time needed to estimate the parameters for all seven groups.

household, both month-to-month and year-to-year statistics can be reported.

Using the CPS as opposed to other publicly available U.S. data such as the Panel Study of Income Dynamics (PSID) has several advantages. First, it is better suited for the study of labor market transitions due to its monthly frequency. Second, it has a large sample size which is particularly important here because, as I explain in the next section, I partition the labor market into disjoint groups and estimate the model separately for each group.

2.4 Segmenting the labor market

The only source of heterogeneity in the model comes from differences in match quality. Indeed, irrespective of worker and firm characteristics, if a pair is of quality n, output will be $f(n, s)$ and the wage bill $w(n, s)$. This model outcome is restrictive and is at odds with a large literature who finds that worker and firm effects are very predictive of wages (see, e.g Abowd et al., 1999, and more recently Bonhomme et al., 2019 and Lentz et al., 2021).

To address this concern, I partition the labor market into groups and estimate the model separately for each group. Partitioning the labor market imposes the counterfactual restriction that there are no flows in and out of the groups since, by construction, the groups are disjoint. However, it relaxes the assumption that the only heterogeneity is in match quality. This assumption now becomes: conditional on n, all workers within a group are identical. In practice, I partition the labor market by workers' educational attainment. I create seven disjoint groups which are described in Table 1. The first group is the set of workers who did not receive a High School diploma or equivalent. This includes workers who do not have any form of schooling up to and including workers who went to High School, but did not complete the 12th grade. The second and largest group (28.2%) corresponds to workers who received a High School diploma or equivalent, but did not pursue a college or professional degree. The third group are workers who attended college but did not receive a degree. The fourth group corresponds to workers who hold an associate's degree from either an occupational, vocational, or academic program. The fifth and second largest group (21.6%) is workers who hold a Bachelor's degree. The sixth group is the set of workers who hold a Master's degree. Finally, the last and smallest group (3.1%) are workers who either received a professional or doctoral degree.

Table 1: Description of the education groups

Group	Descritpion	CPS Code	Frequency (%)
No HS	Up to and including those that didn't complete HS (i.e. no diploma awarded)	2, 10, 20, 30 40, 50, 60, 71	9.3
HS	HS diploma or equivalent	73	28.2
Some college	Some college, but no degree awarded	81	18.9
Associate's	Associate's degree awarded (occupational, vocational, and academic programs)	91, 92	10.4
BA	Bachelor's degree awarded	111	21.6
Master's	Master's degree awarded	123	8.5
Prof/PhD	Professional or doctoral degree awarded	124, 125	3.1

Note: Because of the weighting in the CPS Basic Monthly, the frequencies for the pooled data across all months are the same as the frequencies in each month.

In principle other characteristics or combinations of characteristics could have been used. For example, conditioning not only on educational attainment but also on industry would have been even better since then the assumption is that within an industry and education pair, conditional on match quality, all workers are the same. I do not do this here because the sample size is not large enough. The groups would be too small and there would not be enough variation in the moments to identify the model parameters. Another advantage of using educational attainment is that the flows between the various groups are negligible.

2.5 Data Moments

In this section, I describe which empirical moments are used to estimate the model parameters. The list of moments and their descriptions is presented in Table 2. The first series I compute is market tightness. This is the only series that is not computed for each education group for lack of vacancy posting data by education. To obtain the series, I divide monthly vacancies[8] by the seasonally adjusted monthly unemployment level[9] and use the mean and standard deviations as moments.

Next, I compute the monthly employment-to-employment (EE), employment-to-unemployment (EU), and unemployment-to-employment (UE) transition rates for each education group in the CPS. I follow the methodology of Eeckhout and Lindenlaub (2019) to create these series in the context of the rotation groups structure of the CPS and verify that the resulting series, when aggregated, are similar to the ones for the whole economy[10]. I use the means of each of these series, the standard deviation of the EE rate, and the correlation between the UE and EE rates as moments.

[8] Series JTS000000000000000JOL from the Job Openings and Labor Turnover Survey (JOLTS).

[9] Series LNS13000000 from the Bureau of Labor Statistics (BLS).

[10] I compare the series to the ones computed by Fujita, Moscarini, and Postel-Vinay (2021) and made publicly available (and frequently updated) on the authors' websites.

Table 2: Description of the empirical moments

Description	Moment	Level	Source
Market tightness	$E[\theta_t]$, $\text{sd}(\theta_t)$	aggregate	JOLTS, BLS
EE transition rate	$E[EE_t]$, $\text{sd}(EE_t)$	by education	CPS Basic Monthly
EU transition rate	$E[EU_t]$	by education	CPS Basic Monthly
UE transition rate	$E[UE_t]$	by education	CPS Basic Monthly
	$\text{Cor}(UE_t, EE_t)$		
Unemployment Rate	$E[u_t]$, $\text{sd}(u_t)$	by education	CPS Basic Monthly
u_t^k : fraction of u_t searching for $> k$ months	$E[u_t^3]$	by education	CPS Basic Monthly
Avg. wage	$\text{sd}(\overline{w}_t)$	by education	CPS Basic Monthly
	$\text{Cor}(\overline{w}_t, u_t)$		
Avg. wage after EE	$E[\overline{w}_t^{EE}/\overline{w}_t]$	by education	CPS Basic Monthly
Avg. wage after UE	$E[\overline{w}_t^{UE}/\overline{w}_t]$,$\text{sd}(\overline{w}_t^{UE}/\overline{w}_t)$	by education	CPS Basic Monthly

The unemployment rate is calculated for each education group and I match its mean and standard deviation. I compare these to their BLS counterparts[11] to verify that they are correctly computed. I also compute the fraction of unemployed workers who have been searching for a job for more than 3 months, which I denote as u_t^3, and use its mean as a moment.

Finally, I use the following wage moments[12]. I compute the mean wage and its correlation with unemployment. Next, I compute the average starting wages in the economy following either an EE or an UE transition. I normalize these by the mean wage in the economy and match the means of both and the standard deviation of the UE ratio. In Appendix C.1, I describe how the corresponding model simulated moments are created.

I use the following weights in Equation (12). The mean market tightness and unemployment rate have weight 10. The mean EE, EU, and UE transition rates and the standard deviation of the mean wage have weight 5. The standard deviations of the market tightness and of the unemployment rate have weight 3. All other moments have weight 1.

In total, there are 11 parameters to estimate with 15 moments. Although there is no one-to-one mapping from the moments to the model parameters, I provide here a heuristic identification argument. ϕ is an approximation of the mean wage for a given quality and state. The other loading parameters on the production function, ϕ_2 and ϕ_3, will be pinned down by moments relating to wage dispersion and growth. Since market tightness in this environment reduces to the number of vacancies posted, it will be affected by the exogenous separation rate δ and the cost of posting a vacancy κ. In particular, the standard deviation of vacancies identifies κ. The EE and UE rates, as well as the average wages following either of these transitions, will help identify the parameter on the meeting function, ι. The parameters governing the transition dynamics and magnitude of the aggregate state, $\{p_1, p_2, \text{step}\}$, are identified by variation in

[11] For the unemployment rate, these are the series LNS14027659 (No HS), LNS14027660 (HS), LNS1027689 (Some college or Associate's), LNS14027662 (Bachelor's), CGMD25O (Master's), CGPD25O (Professional), and CGDD25O (PhD).

[12] Throughout the paper, all wage moments are converted to 2019 USD using the CPI-U.

the standard deviation of unemployment and market tightness since these two model objects are very responsive to changes in the productivity level, as I highlight in an impulse response exercise in Section 3.2.

Finally, although δ is identified by variation in other moments as discussed above, it has an almost one-to-one mapping with the mean EU transition rate. When there is no change in the aggregate state, there will be no endogenous separations because matches that were previously feasible will remain so. Hence, the only variation in the EU rate in those periods will be due to exogenous separations. The same argument holds in periods of expansion when the productivity level increases. However, during recessions when the state decreases, it could be that some matches are no longer jointly profitable and so are endogenously separated. In that case, the EU rate will not only be pinned down by δ, but also by fluctuations in the number of matches that are feasible. However, as I discuss later, the estimated transition matrices Π will have a heavy diagonal, making it so that the vast majority of periods do not correspond to a state change and even less to a state decrease. Hence, most of the variation in the EU rate will be due to δ.

2.6 Estimated Parameters

I now briefly discuss the estimated parameters which are presented in Table 3 before turning to model fit. The exogenous separation rate δ is decreasing in educational attainment: workers with higher education levels are more shielded to random separations. The loading parameters on the production function ϕ_1, ϕ_2, ϕ_3 imply that output is increasing in education. Moreover, the flow cost of posting a vacancy κ is roughly constant for all groups, hence posting a vacancy becomes cheaper, relative to output, to firms for the higher education groups. Next, the output relative to home production, a key object in determining which matches are feasible in any given state, is increasing in education. Finally, the Zipf share is approximately constant across groups implying that, for example, there are no significant differences in the probability of creating matches of the highest quality across groups[13].

[13]Note that for $\eta = 0.4$, the probabilities of forming a match of qualities $\{1, 2, \ldots, N\}$, where $N = 10$ are approximately $\{0.175, 0.133, 0.113, 0.101, 0.092, 0.086, 0.081, 0.076, 0.073, 0.070\}$. So the probability of creating a match of the lowest quality is $0.175/0.070 = 2.5$ times greater than the probability of creating a match of the highest quality.

Table 3: Estimated Parameters

	Description	No HS	HS	Some college	Associates	BA	Master's	Prof./PhD
δ	exogenous	.025	.015	.015	.012	.008	.008	.006
	separation rate	(.010)	(.006)	(.006)	(.009)	(.009)	(.016)	(.008)
κ	vacancy	729	614	687	690	752	755	689
	posting cost	(164)	(201)	(196)	(181)	(168)	(175)	(225)
ϕ_1		1154	1154	1463	1755	3026	3396	3198
	Prod. function:	(857)	(1017)	(1161)	(1365)	(2613)	(2158)	(2590)
ϕ_2	$f(n,s) =$	104	119	124	168	236	272	197
	$z(s) \times (\phi_1 + \phi_2 n + \phi_3 n^2)$	(86.8)	(110)	(95)	(143)	(196)	(200)	(155)
ϕ_3		12.2	12.5	14.9	20.4	42.0	53.1	44.2
		(10.7)	(13.6)	(14.1)	(18.9)	(55.3)	(69.1)	(38.0)
step	magnitude of	.045	.044	.041	.042	.049	.043	.038
	prod. shock	(.025)	(.028)	(.024)	(.026)	(.028)	(.029)	(.028)
p_1	prob. moving	.035	.042	.037	.032	.044	.041	.021
	up 1 state	(.026)	(.030)	(.028)	(.031)	(.041)	(.037)	(.019)
p_2	prob. moving	.055	.046	.054	.053	.042	.033	.026
	down 1 state	(.026)	(.030)	(.027)	(.031)	(.028)	(.032)	(.023)
ι	matching	.883	.898	1.00	.966	1.15	.977	1.24
	function	(.448)	(.430)	(.431)	(.399)	(.518)	(.544)	(.412)
γ	home production	548	401	509	553	472	511	592
	$h = z(s) \times \gamma$	(264)	(267)	(279)	(266)	(245)	(256)	(158)
η	Zipf shape	.395	.444	.402	.449	.365	.367	.400
	parameter	(.301)	(.351)	(.294)	(.350)	(.358)	(.381)	(.370)

Note: The standard deviation of the coldest chain is reported in parentheses.

2.7 Model Fit

In Table 4, I draw a series of shocks over 3,000 periods (discarding the first 600 periods for burn-in) using the estimated Π transition matrix and simulate the relevant time series. I report the resulting model simulated moments and compare them to their empirical counterparts for each education group. Recall that the first two moments, the mean and standard deviation of market tightness, are computed for the whole economy but estimated for each education group. Hence, to assess the fit of these moments, it is best to look at the frequency weighted mean across groups[14].

[14]Doing so, note that the estimated mean market tightness is

$$.541 * .093 + .651 * .282 + .570 * .189 + .660 * .104 + .907 * .216 + 1.22 * .085 + .763 * .031 = .733$$

and the mean standard deviation is

$$.152 * .093 + .218 * .282 + .206 * .189 + .256 * .104 + .282 * .216 + .297 * .085 + .276 * .031 = .236$$

where the weights are the frequencies reported in Table 1. So the mean market tightness is overestimated in the model, but the volatility is close to what we see in the data.

Table 4: Targeted moments

	No HS		HS		Some college		Associates		BA		Masters		Prof./PhD	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
$E[\theta]$	.552	.541	.552	.651	.552	.570	.552	.660	.552	.907	.552	1.22	.552	.763
sd(θ)	.288	.152	.288	.218	.288	.206	.288	.256	.288	.282	.288	.297	.288	.276
$E[EE]$	.025	.036	.021	.026	.024	.025	.019	.021	.020	.016	.018	.016	.017	.012
sd(EE)	.007	.005	.005	.004	.006	.004	.005	.004	.004	.002	.005	.001	.005	.002
$E[EU]$	.026	.025	.015	.015	.013	.015	.009	.012	.007	.008	.006	.008	.004	.006
$E[UE]$	.223	.316	.247	.356	.266	.353	.263	.376	.269	.505	.256	.531	.237	.476
Cor(UE,EE)	.585	.875	.753	.848	.741	.849	.609	.821	.683	.772	.494	.744	.117	.710
sd$(\overline{w}_t)$	91.1	245	90.5	358	136	405	131	590	189	1252	203	1272	386	1378
Cor$(\overline{w}_t, u_t)$	-0.427	-0.914	-0.434	-0.894	-0.533	-0.884	-0.357	-0.889	-0.133	-0.885	-0.104	-0.905	-0.153	-0.887
$E[\overline{w}_t^{EE}/\overline{w}_t]$	.933	1.17	.913	1.12	.857	1.10	.917	1.09	.911	.956	.949	.943	.971	.971
$E[\overline{w}_t^{UE}/\overline{w}_t]$	.808	.654	.764	.574	.691	.596	2.00	.557	.682	.481	.687	.461	.643	.486
sd$(\overline{w}_t^{UE}/\overline{w}_t)$	.109	.022	.075	.030	.086	.031	.340	.033	.099	.031	.193	.025	.356	.040
$E[u_t]$	.123	.054	.067	.028	.056	.028	.041	.021	.032	.008	.025	.008	.016	.007
sd(u_t)	.032	.011	.023	.008	.020	.008	.015	.006	.011	.003	.008	.002	.006	.003
$E[u_t^3]$	.446	.324	.472	.275	.478	.279	.400	.253	.443	.131	.413	.109	.476	.160

Note: To compute the time series and corresponding moments, the model is simulated for 3,000 periods (250 years) where the first 600 periods (50 years) are burn-in. To alleviate the effect of initial conditions, I do this 500 times and report the mean of each moment.

The model does a good job in matching the levels of EE, EU, and UE transition rates. Furthermore, it is able to mimic the fact that the EU transition rate is strongly decreasing in education. As previously discussed, this is because the estimated δ are strongle decreasing in education. However, the model predicts that the UE rate is increasing in education, whereas in the data it does not appear to be. Next, volatility of wages is overestimated by the model, but it is does a relatively good job in match relative wages both out of employment and unemployment. Finally, the model underestimates unemployment for all groups. In Appendix C.3, I assess how well the model matches several untargeted moments.

3 Cyclicality of the Average Match Quality

In this section, I use the estimated model to analyze the effects of the business cycle on the average match quality in the economy. I start by discussing the two main channels that impact its cyclicality and then turn to a numerical exercise to help disentangle these two channels. Finally, I explore the relationship between average match quality and mismatch.

3.1 Average Match Quality over the Cycle

The average match quality in period t is

$$\overline{n}_t = \frac{\sum_{n=1}^{N} e_t(n) \times n}{\sum_{n=1}^{N} e_t(n)}.$$

Two mechanisms in the model impact its movements across the cycle. First, during recessions, some matches that were previously feasible are separated. This is the Schumpeterian *cleansing effect* of recessions and it implies an increase in $\overline{n}$. Let

$$n^*(s,e) := \min\{n : S(n,s,e) > 0\}$$

denote the cutoff quality such that all matches below it are not feasible. For example, if $n^* = 5$, then all meetings between workers and firms that yield a quality less that 5 are not formed, and all inherited matches that are less than 5 are separated. The degree to which this threshold fluctuates over time is pinned down by the value of production relative to home production across states. If $f(n,s)$ is much bigger than $h(s)$, n^* will not fluctuate much. On the other hand, if home production is close to output, then movements in the aggregate state can make a previously feasible match infeasible. Second, during recessions, the number of vacancies posted by firms decreases and so the contact rate to the worker, $p(\theta)$, decreases. So workers can no longer sort into better jobs as quickly as before which results in a decrease in $\overline{n}$. This is what Barlevy (2002) calls the *sullying effect*.

In Table 5, I report several statistics of $\overline{n}$. Average match quality is very high for each education group. This is because, in the model, the highest quality matches only separate exogenously and the number of workers willing to accept an offer of quality n is increasing in n. These two features of the model result in heavy bunching at the upper tail of the match distribution. Average match quality is higher and less volatile as educational attainment increases. Furthermore, the correlation between $\overline{n}$ and u is strongly negative across all groups, indicating that average match quality is procyclical. Finally, the correlation between market tightness and unemployment, a measure of the strength of the sullying effect, is also strongly negative for all groups.

Table 5: Cyclicality of the average match quality

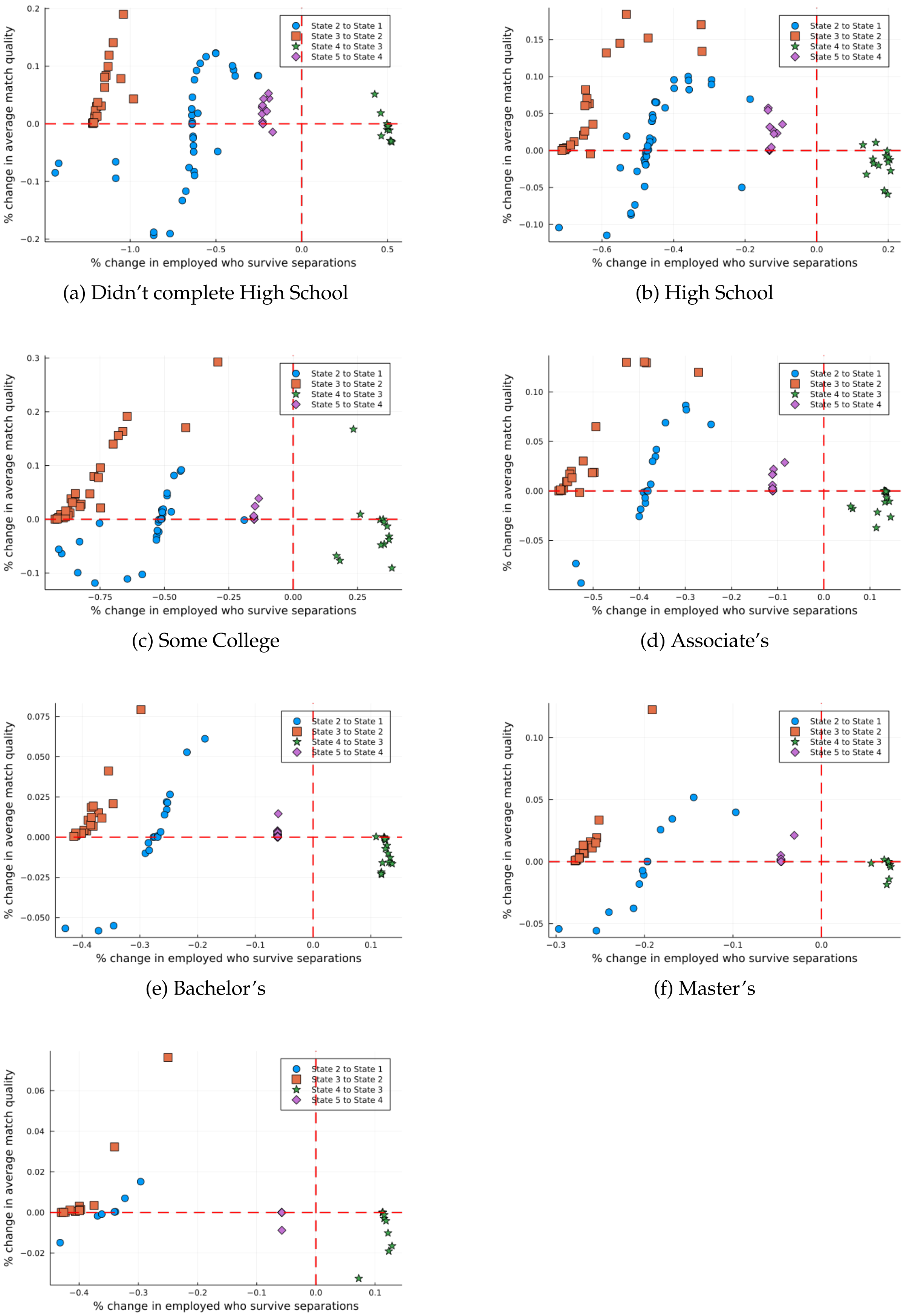
	No HS	HS	Some college	Associate's	B.A.	Master's	Prof./PhD
$E[\overline{n}_t]$	8.50	9.01	9.05	9.21	9.60	9.61	9.67
$\text{sd}(\overline{n}_t)$	.127	.110	.111	.090	.050	.036	.055
$\text{Cor}(\overline{n}_t, u_t)$	-0.803	-0.775	-0.840	-0.754	-0.771	-0.811	-0.789
$\text{Cor}(\theta_t, u_t)$	-0.835	-0.823	-0.806	-0.841	-0.836	-0.865	-0.859

Note: To compute the time series and corresponding moments, the model is simulated for 3,000 periods (250 years) where the first 600 periods (50 years) are burn-in. To alleviate the effect of initial conditions, I do this 500 times and report the mean of each moment.

Although the correlation between $\overline{n}$ and u is informative for the overall cyclicality of average match quality across a large number of periods, it could be masking key dynamics for two main reasons. First, although the model is simulated for 250 years (3,000 periods), since the estimated transition matrices for the aggregate shock Π have a heavy diagonal, the number of periods that follow a state change is small. Most of the time, the state does not change. The number of periods for which there is a decrease in the productivity level, what I refer to as recessions, is even smaller. Second, it could be the case that the magnitude of this correlation is driven by particularly bad recessions. Going from the second lowest state to the worst is intrinsically different than going from the highest state to the second-best. To explore this, I analyze the different impacts on average match quality when conditioning on not only the current state but also on the state in the previous period.

In Figure 2, for each education group, I plot the percent change in $\overline{n}$ against the percent change in the mass of employed workers who survive separations, $\sum_{n=1}^{N} e_+(n)$. I use the latter as a measure of the cycle since it occurs before reallocation and sorting within a period, but includes the initial effect of the state on the endogenous distribution of workers across matches. Finally, I only include in the plots periods for which there has been a decrease in the aggregate state. There are four such cases: when the shock goes from state 2 to state 1, from state 3 to state 2, from state 4 to state 3, and from state 5 to state 4.

Figure 2: Change in average match quality when the state decreases



(a) Didn't complete High School

(b) High School

(c) Some College

(d) Associate's

(e) Bachelor's

(f) Master's

(g) Professional/PhD

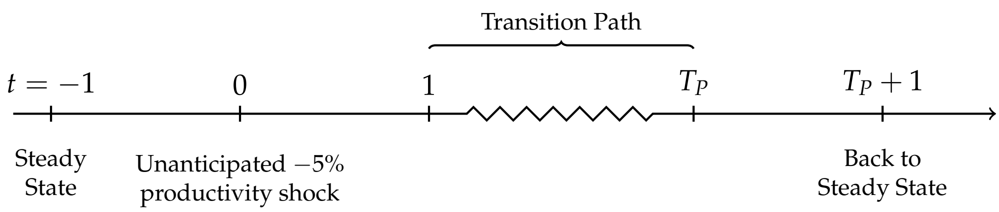
Note: This figure plots the percent change in $\overline{n}$ against the percent change in $\sum_n e_+(n)$. I only plot periods for which there is a decrease in the state. The dashed red lines correspond to 0%.

In periods where the state decreased from 2 to 1, both the sullying and cleansing effects can dominate as there are positive and negative changes in average match quality. This holds across all seven education groups. Next, moves from state 3 to state 2 are concentrated in the top left quadrant for each group, suggesting that the cleansing effect is stronger. They are also associated with larger percent changes in the measure of workers who survive separations. The points corresponding to moves from state to 5 to 4 are also in the top left quadrant, but are concentrated in a fairly small region corresponding to a small increase in average match quality and a small decrease in the mass of workers surviving separations. Finally, the points corresponding to moves from state 4 to 3 are in the lower right quadrant. For all groups, they correspond to an increase in the number of workers surviving separations and a decrease in average match quality. The sullying effect clearly dominates during this last set of periods.

3.2 Impulse Responses to Productivity Shock

In order to further disentangle the effects of the cleansing and sullying effects on the average match quality, I conduct the following exercise, illustrated in Figure 3. Suppose the economy is in steady state at the highest level of productivity ($s = s_5$) in period $t = -1$. Then an unanticipated and one-period -5% productivity shock hits the economy and lasts for T_P periods, after which the economy returns to steady state at $s = s_5$ in period $T_P + 1$. This thought exercise consists in two key assumptions. First, the shock is not persistent. Second, agents in the economy have perfect foresight. They know the shock is not persistent and that the economy will return to its steady state in period $T_P + 1$. This implies that there is no uncertainty since agents have full information on the path of s after the shock occurs (before it occurs they do not since the shock is unanticipated).

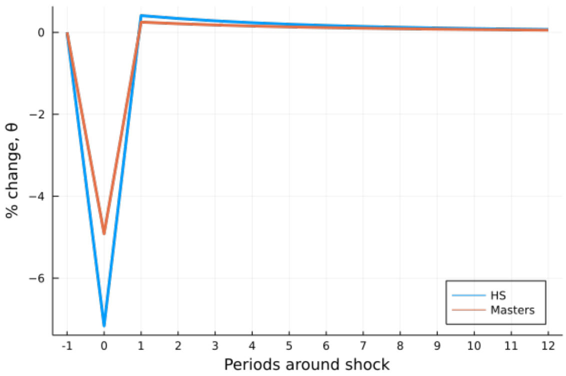
Figure 3: Illustration of the timing of the productivity shock



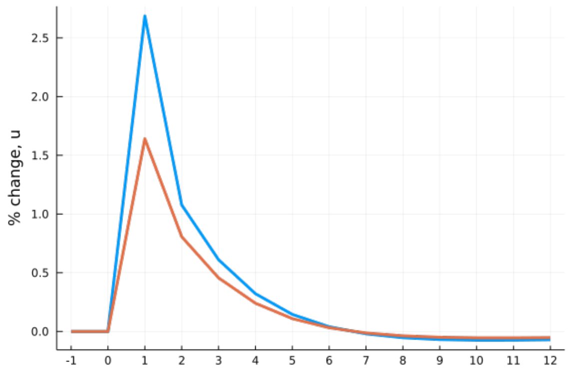
The impulse responses of market tightness, unemployment, hires (both out of unemployment and out of employment), and average match quality are graphed in Figure 4 for two education groups (High School and Master's). The numerical procedure used to solve for the transition path back to steady state is relegated to Appendix B.2. Right after the shock hits, some matches that were previously feasible are no longer jointly profitable and endogenously separate. This is the cleansing effect and results in a decrease in the mass of workers who survive separations, i.e. $e_{0+} \leq e_{-1+}$. Equivalently, the pool of workers who are unemployed following separations increases so that $u_{0+} \geq u_{-1+}$. Next, firms decreases their vacancy posting as shown Figure 4a. This decrease in market tightness results in a decrease in the contact rate to the worker, but an increase in the contact rate to the firm. Together, they translate to a decrease in meetings between workers and firms and so a decrease in hiring, as shown in

Figure 4c. At the end of the period, after the new matches have formed and production has occurred, unemployment is higher than it was in the previous period ($u_0 > u_{-1}$) as can be seen in Figure 4b.

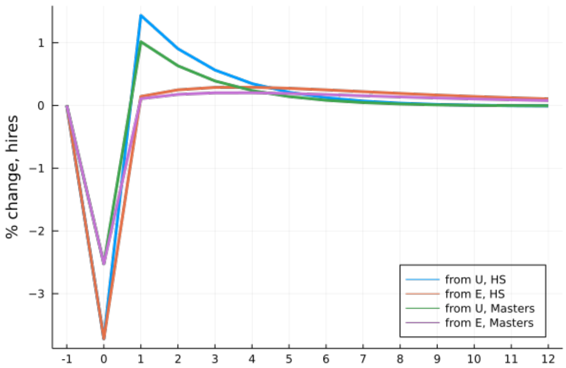
Figure 4: Impulse Responses to a -5% productivity shock



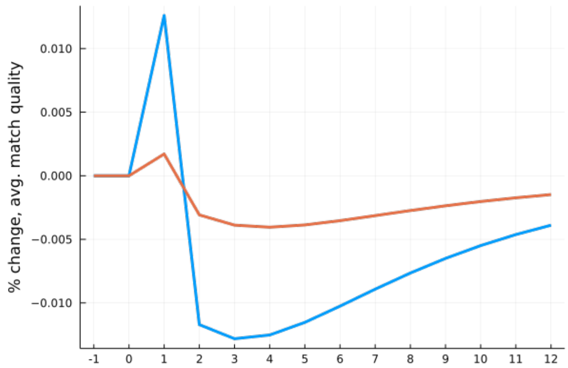
(a) Market Tightness



(b) Unemployment



(c) Hires (UE and EE)



(d) Average Match Quality

Note: To facilitate comparisons between these two groups, I set $\eta = 0$, which makes it so that the probability that a meeting between a worker and firm yields a quality n is $1/N$, $\forall n$.

Since the productivity shock only affects endogenous separations, average match quality increases in the period the shock hits as depicted in Figure 4b. Note that the increase is small in magnitude because in the model there is significant mass in the upper tail of the match quality distribution. To see this, recall that the mass of workers who would accept to form a match of quality n is $u_+ + \sum_{k=1}^{n-1} e_{k+}$ and is increasing in n. Indeed, it is not profitable for the workers in the best quality matches to move down the job ladder and accept offers of a lower quality. Furthermore, these matches can only separate exogenously[15] which occurs with low probability. All these factors make it so there is significant bunching at the top end of distribution, and this is especially true in the stationary version of the model.

The next period, at $t = 1$, the economy is back at the highest productivity level and so firms

[15]In principle, the highest quality matches can separate endogenously in the case where the state of the economy is so bad that no matches are feasible, but this is not a realistic case nor is it one I have encountered in any of the model parameterizations.

take advantage of the larger pool of unemployed workers by posting more vacancies. In fact, there is a slight over adjustment and $\theta_1 > \theta_{-1}$. This translates to a large increase in hiring from unemployment (more than 1% above its steady state level for the group of workers with only a High School diploma or equivalent) and from employment. On average, matches formed out of unemployment will be of lower quality than matches formed out of employment so this results in a decrease in the average match quality in period $t = 1$. This is the sullying effect. In the following periods, the series plotted all converge back to their steady state values at varying speeds. Market tightness recovers the fastest and average match quality the slowest.

Finally, the mechanisms described above are identical across the two education groups. The only difference is in the magnitude of the responses. The depth of unemployment, decrease in vacancy posting by firms, and decrease in hiring, both out of unemployment and out of employment, is greater for the High School group. These translate to a larger initial increase of the average match quality relative to that of the Master's group. However, the response in hiring is larger for the High School group and so the sullying effect is bigger. These differences in magnitude across the two groups primarily stem from differences in the relative value of output and home production. This ratio is governed by the parameters $\{\phi_1, \phi_2, \phi_3, \gamma\}$ and is strictly increasing in educational attainment for all match qualities.

3.3 Mismatch over the cycle

Finally, in this section, I explore the relationship between average match quality and mismatch. In the model, a Social Planner maximizing output would allocate all workers to the firms with which they are best matched, i.e. all matches would be of quality N. I assess the level of mismatch in the economy, how far away from this efficient allocation, using three measures. First, I look at the fraction of the employed who are mismatched, which I denote as

$$M_t^1 := \frac{\sum_{n=1}^{N-1} e_t(n)}{\sum_{n=1}^{N} e_t(n)}.$$

Next, I look at a measure of aggregate distance from the optimal match defined as

$$M_t^2 := \frac{\sum_{n=1}^{N-1} e_t(n) \times (N-n)}{\sum_{n=1}^{N} e_t(n) \times N}.$$

The idea here is that, all else equal, it would take longer for a worker in a match of quality $n = 1$ to move up the job ladder to the best match than it would for workers in higher quality matches $k > 1$. To capture this, the measures of workers in each match quality no longer have the same weight, as is the case in M_t^1, but are weighted by the distance to the best match quality. I normalize by $\sum_n e_t(n) \times N$ so M^2 can be interpreted as the relative distance to the efficient allocation. Finally, the last measure captures the losses in surplus due to mismatch:

$$M_t^3 := \frac{\sum_{n=1}^{N-1} e_t(n) \times \left[S(N, s_t, e_t) - S(n, s_t, e_t)\right]}{\sum_{n=1}^{N} e_t(n) \times S(n, s_t, e_t)}.$$

This measures how much higher the surplus of the mismatched would be if all matches were optimal as a fraction of total surplus in the economy. Since the joint surplus is increasing in match quality, here higher quality matches have a higher weight than lower quality matches, as opposed to M_t^2.

Notice that, in this setting, the level of mismatch, as measured by any of the above three statistics, is perfectly equivalent to the level of efficient matches. For example, the fraction of workers perfectly matched is $e(N)/\sum_n e(n) = 1 - M_1$ and a similar relationship holds for the two other measures. I focus on mismatch because the object of interest in this study is the average match quality and it is more natural to think of how it relates to mismatch.

In Table 6, I report several statistics for each of these three measures of mismatch. First, note that there is significant mismatch in the model across the business cycle. The fraction of workers in inefficient matches (M^1) is above 14% for all groups and more than 50% for the group of workers that didn't complete High School. Similarly, the surplus losses due to mismatch (M^3) are high. In contrast, M^2 is much smaller than the two other measure of mismatch. This is because the distribution of workers across matches in the model has a fat right tail. Next, all three measures have a strong positive correlation with unemployment and a strong negative correlation with average match quality. Finally, there is a clear relationship with educational attainment. All three measures of mismatch, as well as the magnitudes of their correlation with unemployment, are decreasing in education. However, for the correlations with average match quality, there does not appear to be any relationship with education.

Table 6: Summary statistics for the measures of mismatch

	No HS	HS	Some college	Associate's	B.A.	Master's	Prof./PhD
$E[M_t^1]$	.509	.369	.357	.306	.168	.166	.143
$\mathrm{sd}(M_t^1)$	.028	.029	.027	.027	.017	.013	.020
$\mathrm{Cor}(M_t^1, u_t)$	.697	.645	.620	.620	.644	.700	.684
$\mathrm{Cor}(M_t^1, \overline{n_t})$	-0.967	-0.960	-0.958	-0.957	-0.962	-0.967	-0.969
$E[M_t^2]$	.149	.099	.094	.078	.040	.039	.034
$\mathrm{sd}(M_t^2)$	.013	.011	.011	.009	.005	.004	.005
$\mathrm{Cor}(M_t^2, u_t)$	.807	.773	.744	.758	.769	.813	.789
$\mathrm{Cor}(M_t^2, \overline{n_t})$	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
$E[M_t^3]$	.526	.361	.343	.283	.145	.144	.121
$\mathrm{sd}(M_t^3)$	.048	.040	.040	.032	.016	.012	.017
$\mathrm{Cor}(M_t^3, u_t)$	.717	.671	.642	.648	.660	.701	.675
$\mathrm{Cor}(M_t^3, \overline{n_t})$	-0.942	-0.944	-0.961	-0.963	-0.966	-0.951	-0.970

Note: To compute the time series and corresponding moments, the model is simulated for 3,000 periods (250 years) where the first 600 periods (50 years) are burn-in. To alleviate the effect of initial conditions, I do this 500 times and report the mean of each moment.

This result is the opposite of the one found in Baley, Figueiredo, and Ulbricht (2021) who conclude that mismatch is procyclical. As in this model, their framework allows for the cleansing and sullying effects to both operate jointly, but they find that the cleansing effect strictly dominates.

4 Minimum Wage

In this section, I extend the model to allow for an exogenous minimum wage and show that not taking into account the cyclicality of the average match quality can lead to miscalculating the effects of the policy. First, I present how the model is modified to allow for an exogenous minimum wage. Then, I discuss how the presence of the minimum wage affects agents' behavior within the model and how it relates to the cyclicality of the average match quality. Finally, I assess the effects of the policy in both the stationary and stochastic versions of the model.

4.1 Adding a minimum wage

So far, the model described can be thought of as a framework where there is no legal minimum wage. Extending the model to include a minimum wage entails adding an additional constraint to the Nash bargaining problem. As in Flinn (2006), the surplus splitting rule now becomes

$$w(n,s,e) = \underset{w \geq w_{\min}}{\arg\max} \left[W(n,s,e) - U(s,e)\right]^{\alpha} J(n,s,e)^{1-\alpha}$$

where $w_{\min}$ denotes an exogenously set minimum wage. I assume that $w_{\min}$ applies to all potential matches and is time invariant[16].

The impact of this added constraint can best be understood by thinking of the cutoff match quality $\hat{n}(s,e)$ such that all $n < \hat{n}(s,e)$ are paid the minimum wage. This threshold is defined as

$$\hat{n}(s,e) := \max\{n : w_{\min} \geq w(n,s,e)\}.$$

The added constraint to the wage bargaining problem is only binding in a given state (s,e) if $\hat{n}(s,e) \geq n^*(s,e)$. In that case, all matches of quality $n \in \{n^*(s,e), \ldots, \hat{n}(s,e)\}$ would be feasible in the absence of $w_{\min}$, but would yield a lower wage than $w_{\min}$. In the presence of $w_{\min}$, these matches are consummated and the workers are paid $w_{\min}$ if the firm's participation constraint is still satisfied at $w_{\min}$, i.e. if the firm surplus under this higher wage bill remains positive. Note that if $\hat{n}(s,e) = n^*(s,e)$, only the lowest feasible quality match gets paid $w_{\min}$. On the other hand, when $\hat{n}(s,e) < n^*(s,e)$, the minimum wage is not feasible since $S(\hat{n}(s,e),s,e) < 0$, and so it does not impact allocations in the economy. I focus on the case where $w_{\min}$ is binding, i.e. when $\hat{n}(s,e) \geq n^*(s,e)$. In practice, this entails choosing $w_{\min}$ to be large enough to impact wages paid in the economy. I set $w_{\min}$ to $2,400, which corresponds to a worker earning an hourly wage of $15 over the course of a month[17].

Value Functions. The worker and firm value functions (and so the joint surplus of a match) now depend on the cutoff $\hat{n}(s,e)$ which is itself a function of $w_{\min}$. This implies that the threshold n^* now also depends on $w_{\min}$. In what follows, I continue to write this cutoff as $n^*(s,e)$,

[16] The empirical minimum wage varies across time because of inflation. However, note that the empirical moments I match are all normalized to 2019 USD so this is not a concern.

[17] I assume the worker works all four weeks for 40 hours per week.

but note that it should really be written as $n^*(s,e;w_{\min})$ to highlight the fact that, in any given period, this cutoff in the presence of $w_{\min}$ is not necessarily the same as what it would be if there was no binding minimum wage. First, note that the value of unemployment is now

$$\begin{aligned} U(s,e;w_{\min}) = h(s) + \beta \mathbf{E}_{s'|s}\Big[& U(s',e';w_{\min}) \\ & + p(\theta(s,e_+)) \sum_{j=1}^{\hat{n}(s',e')} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{W(\hat{n}(s',e'),s',e') - U(s',e';w_{\min}),0\} \\ & + p(\theta(s,e_+)) \sum_{j=\hat{n}(s',e')+1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{W(j,s',e') - U(s',e';w_{\min}),0\}\Big]. \end{aligned} \tag{13}$$

The transitions into employment from unemployment can be separated into two cases. If an unemployed worker meets with a firm to create a match quality weakly less than $\hat{n}(s',e')$, they accept if it is more profitable than staying unemployed and receives the value associated with earning $w_{\min}$ next period, $W(\hat{n}(s',e'),s',e')$. On the other hand, if the match quality is strictly greater than $\hat{n}(s',e')$ and it is jointly profitable to consummate it, then they earn the market wage next period.

For $n > \hat{n}(s,e)$, the value to a worker of employment in a match of quality n is still given by Equation (6) except that the value of unemployment now depends on $w_{\min}$. However, workers in matches of quality $n \in \{n^*(s,e),\ldots,\hat{n}(s,e)\}$[18] now earn $w_{\min}$. So Equation (6) is modified so that the value of being employed in a match of quality n in the presence of a binding minimum wage is given by

$$\begin{aligned} W(n,s,e;w_{\min}) &= 1\{n \le \hat{n}(s,e)\}w_{\min} + \big(1 - 1\{n \le \hat{n}(s,e)\}\big)w(n,s,e) \\ &+ \beta \mathbf{E}_{s'|s}\Bigg[U(s',e') + (1-\delta)1\{S(n,s',e') \ge 0\}\Big(W(n,s',e';w_{\min}) - U(s',e';w_{\min})\Big) \\ &+ (1-\delta)1\{S(n,s',e') \ge 0\} p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{W(j,s',e';w_{\min}) - W(n,s',e';w_{\min}),0\}\Bigg]. \end{aligned}$$

Similarly, the value to the firm now also depends on $w_{\min}$:

$$\begin{aligned} J(n,s,e;w_{\min}) = f(n,s) &- 1\{n \le \hat{n}(s,e)\}w_{\min} - \big(1 - 1\{n \le \hat{n}(s,e)\}\big)w(n,s,e) \\ &+ \beta \mathbf{E}_{s'|s}\left[(1-\delta)1\{S(n,s',e';w_{\min}) \ge 0\}\left(1 - p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}}\right) J(n,s',e';w_{\min})\right]. \end{aligned}$$

[18] Note the value functions are ex post, i.e. they correspond to the value to the worker and to the firm of a formed match, so are only defined for $n > n^*(s,e)$.

In Appendix A.4, I show that the joint surplus is

$$\begin{aligned}
S(n,s,e;w_{\min}) = f(n,s) - h(s) + \beta \mathbf{E}_{s'|s}\Bigg[&(1-\delta)\max\{S(n,s',e';w_{\min}),0\}\left(1 - p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right) \\
&+ \alpha(1-\delta)1\{S(n,s',e';w_{\min}) \geq 0\}p(\theta(s,e_+))\sum_{j=n+1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}S(j,s',e';w_{\min}) \\
&- p(\theta(s,e_+))\alpha\max\{S(\hat{n}(s',e'),s',e';w_{\min}),0\}\sum_{j=1}^{\hat{n}(s',e')}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}} \\
&- p(\theta(s,e_+))\alpha\sum_{j=\hat{n}(s',e')+1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e';w_{\min}),0\}\Bigg].
\end{aligned} \tag{14}$$

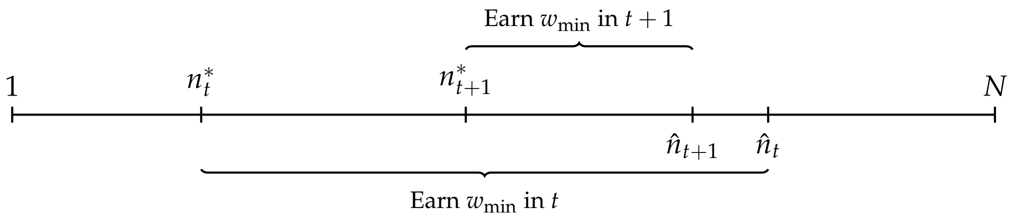
The only difference with Equation (9) is the last two terms in the expectation. The first corresponds to the foregone surplus losses from searching while unemployed and matching with firms to create match qualities that would pay the minimum wage. The second are the same foregone surplus losses, but for matches that would pay a market wage larger than the minimum wage.

4.2 The relationship between $\hat{n}$ and n^*

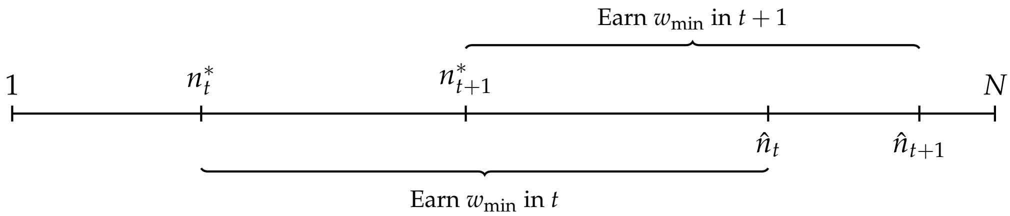
Although the value functions are identical to what we had before except for the wage and the value of unemployment, there are now additional contingencies that merit discussion. Consider workers in matches of quality $n \in \{n^*(s,e),\dots,\hat{n}(s,e)\}$ and suppose, for the sake of argument, that $n^*(s,e) = n^*(s',e')$. If $\hat{n}(s',e') > \hat{n}(s,e)$, these workers still receive $w_{\min}$ next period but the mass of workers earning the minimum wage has increased since it now contains those with match qualities belonging in the set $\{\hat{n}(s,e),\dots,\hat{n}(s',e')\}$. Conversely, if the threshold decreases so that $\hat{n}(s',e') < \hat{n}(s,e)$, then these workers who previously were earning the minimum wage will now be earning the market wage $w(n,s',e') > w_{\min}$.

Next, suppose we restrict $\hat{n}$ to be constant across time. Then if n^* decreases from one period to the next, so that $n^*(s',e') < n^*(s,e)$, all matches of quality $n \in \{n^*(s',e'),\dots,n^*(s,e)\}$ were previously infeasible but now pay $w_{\min}$. If n^* increases, so that $n^*(s',e') > n^*(s,e)$, all the matches $n \in \{n^*(s,e),\dots,n^*(s',e')\}$ are destroyed because they are no longer feasible.

Figure 5: Two of the possible cases when n^* increases



(a) One illustration of $\hat{n}$ decreasing



(b) One illustration of $\hat{n}$ increasing

Finally, if we now allow both thresholds n^* and $\hat{n}$ to vary over time, as they do in the model, with the only constraint being that w_{min} is binding, i.e. $\hat{n} \geq n^*$ in all periods, then there are a lot more cases to consider. The effect on total employment and average match quality becomes more ambiguous. For example, if during a recession, the cleansing effect dominates and the lowest matches are destroyed, i.e. $n^*(s', e') > n^*(s, e)$, and the minimum wage cutoff also decreases so that $n^*(s, e) < n^*(s', e') < \hat{n}(s', e') < \hat{n}(s, e)$ than there are now less matches in (s', e') paying w_{min} then there were in the previous period. However, depending on how the distribution of workers across firms has changed from e to e', this could lead to an increase or a decrease in total employment and average match quality. This scenario is illustrated in Figure 5a. On the other hand, suppose the cleansing effect still holds but now the minimum wage cutoff increases from one period to the next so that $n^*(s, e) < n^*(s', e') < \hat{n}(s, e) < \hat{n}(s', e')$ as illustrated in Figure 5b. Now, it is not clear if there are more or less matches that pay w_{min} and, as in the previous case, what the impacts on total employment and average match quality are. Note that there are more cases to consider in the event that $n^*(s', e') > n^*(s, e)$, but the reasoning is the same so I omit them here. Similarly, there are several other cases to consider if we now suppose $n^*(s', e') < n^*(s, e)$.

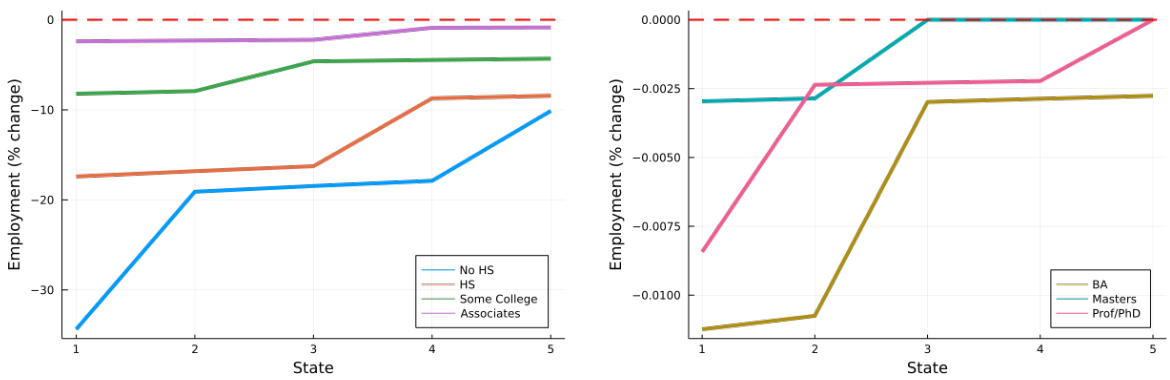
All these cases fundamentally stem from the model being out of steady state, which is, in the context of analyzing the effects of a minimum wage policy, the main difference between this setting and that of Flinn (2006). In a steady state model of the labor market, the thresholds $n*$ and $\hat{n}$ are constant across time. Here, their dynamics crucially determine which allocations are destroyed and created in the economy. In Section 4.3, I shut down aggregate risk and analyze the effects of the minimum wage in the stationary version of the model.

4.3 Binding minimum wage in the stationary environment

Before studying the effects of adding a minimum wage policy to the full model, I analyze its effects on the stationary environment to build intuition. The stationary environment also serves as the benchmark to which I later compare the full model which has average match quality varying across the business cycle. In Appendix B.3, I describe the procedure to solve for the equilibrium outcomes in the presence of $w_{\min}$ for a given state. The approach consists of first solving for the steady state equilibrium in the absence of $w_{\min}$, then computing the minimum wage cutoff $\hat{n}$, and finally using the new value functions, which depend on $\hat{n}$, to check whether the participation constraint is met for matches $n < \hat{n}$. I use the estimated model parameters to do this for each education group[19].

In Figure 6, I graph the percent change in total employment by productivity level and education. The model exhibits an employment effect: total employment decreases in the presence of an exogenous minimum wage policy across all states and education groups. Regardless of educational attainment, the magnitude of this decreased employment is weakly increasing in the state. As the productivity shock decreases in magnitude, employment gets closer to what it would be in the absence of $w_{\min}$. Moreover, the employment effect is decreasing in educational attainment. The loss in employment for workers with more than a Bachelor's degree is almost always less than a tenth of a percent whereas it is almost always greater than ten percent for workers with no more than a High School diploma.

Figure 6: Change in total employment

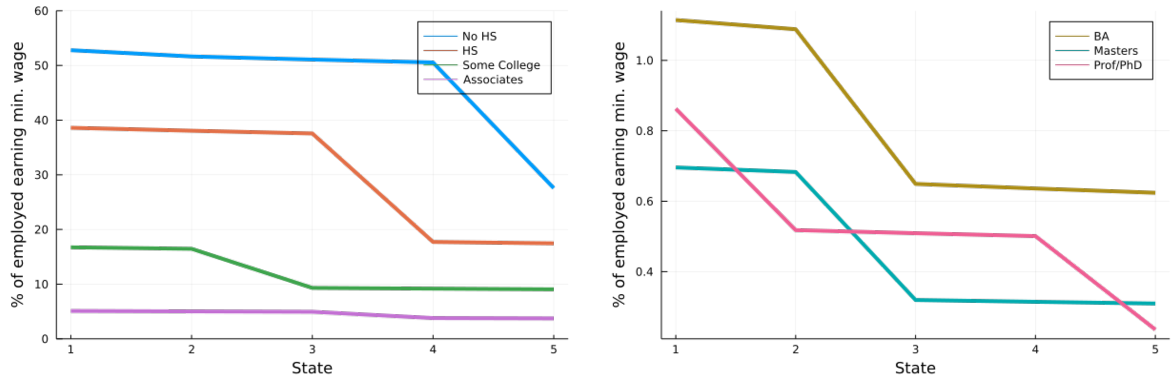


Note: The y-axis corresponds to percent changes of the steady state level of employment under a binding minimum wage $w_{\min} = 2,400\$$ relative to steady state employment without $w_{\min}$. I condition on the state and educational attainment.

Next, I present the fraction of the employed workers who earn $w_{\min}$ by state and educational attainment in Figure 7. Almost less then one percent of the most educated workers earn the minimum wage. On the other hand, for workers with an Associate's degree or less, there is significant bunching at $w_{\min}$. For all education groups, the fraction of employed workers receiving $w_{\min}$ is weakly decreasing in the aggregate state of the economy. This is a feature of the model since wages are increasing in the value of the productivity shock.

[19]Note that the magnitude of the productivity shocks differ by education group since it is governed by the model parameter step. However, it is relatively constant across these groups (see Table 3), which allows us to still make meaningful comparisons.

Figure 7: Percent of employed earning $w_{\min}$ by state and educational attainment



Note: The y-axis corresponds to the percent of the the employed who earn $w_{\min}$\$. As before, I condition on the state and educational attainment.

To disentangle the different channels that lead to a decrease in employment, in Table 7, for both the highest and the lowest productivity level, I present how key model objects differ in the presence of $w_{\min}$. The average match quality increases for workers who have less than an Associate's degree, i.e. those most affected by the introduction of $w_{\min}$. However, the percent of the employed workers who are in the best matches decreases for these groups. This decrease in $e(N)$, all else held equal, would imply a decrease in average match quality. To understand why average match quality still increases, note that unemployment significantly increases, and especially so in the lower education groups. This increase in unemployment is mostly coming from lower quality matches separating since the competitive wage in the absence of $w_{\min}$ is smaller than $w_{\min}$ for these matches. To see this, I compute the change in the percentage of employed worker who are in the lowest half of the distribution (rows 3-4). This group of workers significantly decreases after the presence of $w_{\min}$. Finally, vacancy posting is also lower in the presence of $w_{\min}$ so the rate at which workers sort into better matches is lower. Putting all this together, although the fraction of workers in the best matches decreases, the even larger decrease in the fraction of workers in the lower quality matches dominates and so the average match quality increases.

For the higher education groups, i.e. workers who have a Bachelor's degree or more, the average match quality is almost unchanged. This is because there is almost no change in composition: the distribution of workers across matches is roughly the same as it was in the version without $w_{\min}$. The decreases in unemployment and market tightness are also much smaller. However, these groups see the largest increases in average wages.

Table 7: Change relative to the unconstrained steady state in the highest and lowest state

	No HS	HS	Some college	Associate's	B.A.	Master's	Prof./PhD
$e(N)/\sum_n e(n)$ (high)	-2.11	-1.22	-0.66	-0.33	-0.02	+0.00	+0.00
$e(N)/\sum_n e(n)$ (low)	-3.86	-1.92	-1.36	-0.26	-0.10	-0.03	-0.08
$\sum_{n=1}^{5} e(n)/\sum_n e(n)$ (high)	-5.53	-2.80	-2.93	-0.52	+0.04	-1.05	-1.64
$\sum_{n=1}^{4} e(n)/\sum_n e(n)$ (low)	-3.78	-2.22	-2.72	-1.69	+0.02	+0.00	+0.01
$\overline{n}$ (high)	+2.00%	+1.55%	+1.17%	+0.34%	-0.01 %	+0.00%	+0.00%
$\overline{n}$ (low)	+2.93%	+1.91%	+1.49%	+0.90%	-0.03%	-0.01%	-0.02%
u (high)	+236%	+359%	+211%	+57.7%	+0.40%	+0.00%	+0.00%
u (low)	+680%	+637%	+347%	+141%	+1.36%	+0.36%	+1.39%
θ (high)	-35.5%	-35.2%	-21.9%	-7.75%	-0.36%	+0.00%	+0.00 %
θ (low)	-67.1%	-51.6%	-33.6%	-13.9%	-1.24%	-0.36 %	-1.20%
$\overline{w}$ (high)	+759%	+1137%	+1096%	+1253%	+1341%	+1361 %	+1476%
$\overline{w}$ (low)	+760%	+1172%	+1127%	+1308%	+1423%	+1426%	+1549%

Note: In the first two rows, I report the change in the percentage of workers who are in the best quality matches for the highest and lowest states. For example, for the group of workers who did not complete High School, the percent of employed who are in the best match qualities decreases by 2.11 in the presence of a binding minimum wage and when the aggregate state is 5. I also do the same for the fraction of workers in the lowest 4 match qualities (rows 3-4). Finally, in the other rows, I present the percent changes in average match quality, unemployment, market tightness, and average wage relative to their steady state values in the absence of a minimum wage.

4.4 Binding minimum wage in the stochastic environment

I now return to analyzing the effects of a binding minimum wage in the full model with aggregate risk. As in the previous section, the procedure to solve for the equilibrium is to begin by solving for the equilibrium objects in the absence of $w_{\min}$. Then, compute the sequence of thresholds $\{\hat{n}_t\}_{t=0}^{\infty}$ and verify whether or not the matches in the set $\{n_t^*, \ldots, \hat{n}_t\}$ are feasible under $w_{\min}$. These matches are feasible if and only if their surplus, given by Equation (14), is positive. Computationally, this is a difficult problem because the surplus in any given period depends not only on the future surpluses of all the other match qualities and the distribution of workers across matches, as in the environment without a minimum wage, but in addition it also depends on $\hat{n}$ next period. Because of this, approximation methods, and in particular the collocation method which I use to solve for the equilibrium in the absence of $w_{\min}$, are not applicable. I am currently in the process of developing a solution method for this third step.

To avoid this issue, for now I assume that all matches in $\{n_t^*, \ldots, \hat{n}_t\}$ are consummated. Flinn (2006) makes this same assumption but in his setting, the firm surplus is directly proportional to the match value less the wage so it is less restrictive. The cost of this assumption is that I can not assess the employment effects of the minimum wage policy since the distribution of workers across matches remains the same. However, in this section I analyze the cyclicality of $\hat{n}$, which is closely related to the cyclicality of the average match quality, and argue that even without the employment effects, allowing for aggregate risk captures more of the key mechanisms of the policy.

Table 8: Cyclicality of $\hat{n}$

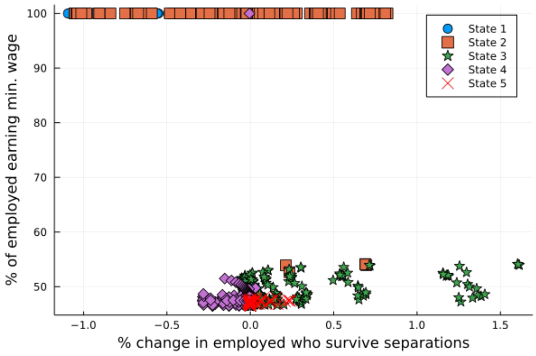
	No HS	HS	Some college	Associate's	B.A.	Master's	Prof./PhD
$E[\hat{n}_t]$	9.66	9.08	8.43	7.03	2.60	1.42	2.54
$\text{sd}(\hat{n}_t)$	.471	.850	.864	.932	.790	.675	.533
$\text{Cor}(\hat{n}_t, u_t)$	.842	.805	.803	.884	.866	.897	.837
$\text{Cor}(\hat{n}_t, \overline{n_t})$	-0.776	-0.672	-0.695	-0.697	-0.812	-0.856	-0.788
Fraction binding	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: To compute the time series and corresponding moments, the model is simulated for 3,000 periods (250 years) where the first 600 periods (50 years) are burn-in. To alleviate the effect of initial conditions, I do this 500 times and report the mean of each moment.

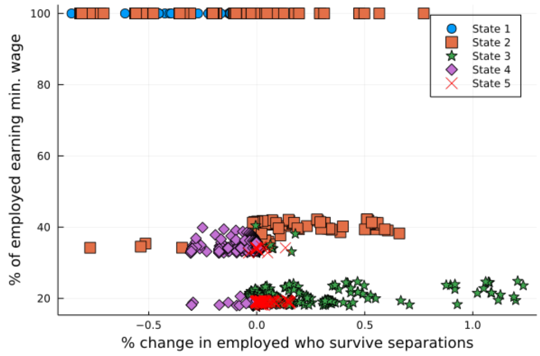
Table 8 presents the mean and standard deviation of $\hat{n}$, its correlation with unemployment and average match quality, and the fraction of the time w_{min} is binding for each education group. By construction, the minimum wage is always binding. The number of match types for which the minimum wage is greater than the competitive wage is decreasing in educational attainment. The minimum wage cutoff is strongly countercyclical: as aggregate conditions worsen and unemployment increases, so does the cutoff meaning that more match types earn w_{min}. Finally, $\overline{n}$ and $\hat{n}$ are strongly negatively correlated: as the minimum wage cutoff increases, so more match qualities are at w_{min}, the average match quality decreases. As discussed in Section 4.2, to correctly understand the effect on the distribution of workers across matches, we would need to see how n^* and $\hat{n}$ vary together from one period to the next.

In Figure 8, I plot the percent of the employed earning w_{min} against the percent change in the number of workers who survive separations and condition on the aggregate productivity level and educational attainment. First, notice that the percent earning w_{min} is roughly decreasing in the state and this relationship holds regardless of educational attainment. Next, note that across groups, the percentage is also decreasing. Indeed, for workers with less than an Associate's degree, in the lowest states, there are periods where everyone employed is earning w_{min}. On the other hand, for the higher education groups, the magnitudes are much smaller: even in state 1, the fraction earning w_{min} does not exceed 3%. Finally, in the best state, the percent earning w_{min} is lowest and is associated with very small positive changes in the number of workers who survive separations. In the second best state, there are slightly more workers earning w_{min}, especially in the lower education groups, but these states are associated with negative percent changes in the number of workers who survive separations.

Figure 8: Percent of employed earning the minimum wage

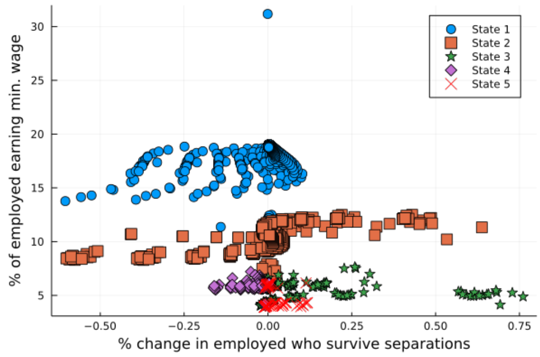


(a) Didn't complete High School



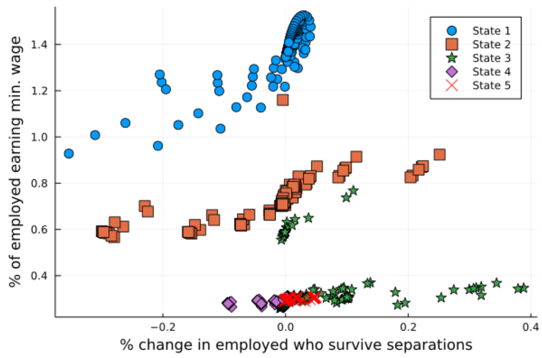
(b) High School

(c) Some College



(d) Associate's

(e) Bachelor's



(f) Master's

(g) Professional/PhD

Note: This figure plots the percent of workers earning $\underline{w}$ against the percent change in $\sum_n e_+(n)$. As in Figure 2, I use the latter as a measure of the business cycle. I condition on education and the aggregate state.

This figure illustrates that the percent of workers earning $w_{\min}$ varies considerably depending on the state of the economy. Hence, restricting the setting to be stationary, as is done in Section 4.3, omits by construction a crucial channel by which employment is affected by the presence of a binding minimum wage.

5 Conclusion

In this paper, I study a general equilibrium model of the labor market with heterogeneous match qualities, on-the-job search, and aggregate risk. I estimate the model separately by worker education using U.S. monthly data from December 2001 to December 2019. I find that the average quality of an employer-employee match is procyclical and explore the interaction between the cleansing and sullying effects. Finally, I extend the model to include a binding minimum wage and study its effects in both the stationary and stochastic versions of the model.

In future work, I plan to complement the analysis conducted in this paper by using French matched employer-employee administrative data. These data would allow me to provide reduced-form evidence on the cyclicality of the average match quality and to incorporate richer worker and firm characteristics in the analysis and estimation.

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A Model Appendix

This appendix contains derivations and proofs omitted in the main text. First, in Appendix A.1, I derive the main expression for the joint surplus, Equation (9). Next, in Appendix A.2, I solve the Nash bargaining problem to derive the wage equation. In Appendix A.3, I prove Proposition 1. Finally, in Appendix A.4, I describe how to solve for the equilibrium outcomes when an exogenous minimum wage is present.

A.1 Derivation of Joint Surplus

Plugging Equation (7), Equation (6), and Equation (5) into the definition of the joint surplus and simplifying yields

$$
\begin{aligned}
S(n,s,e) &:= W(n,s,e) - U(s,e) + J(n,s,e) \\
&= f(n,s) - h(s) + \beta \mathbf{E}_{s'|s}\Bigg[(1-\delta)1\{S(n,s',e') \geq 0\}\Big(W(n,s',e') - U(s',e')\Big) \\
&\quad + (1-\delta)1\{S(n,s',e') \geq 0\} p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{W(j,s',e') - W(n,s',e'), 0\} \\
&\quad + (1-\delta)1\{S(n,s',e') \geq 0\}\left(1 - p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}}\right) J(n,s',e') \\
&\quad - p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{W(j,s',e') - U(s',e'), 0\}\Bigg]
\end{aligned}
$$

Next, using Equation (11), the identity $\max\{a-b, 0\} = \max\{a,b\} - b$, and the identity $1\{x \geq 0\} \times x = \max\{x, 0\}$, the expression for total surplus can be rewritten as

$$
\begin{aligned}
S(n,s,e) = f(n,s) - h(s) + \beta \mathbf{E}_{s'|s}\Bigg[\\
&(1-\delta)\max\{S(n,s',e'),0\}\Bigg(1 - \alpha p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \\
&- (1-\alpha) p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}}\Bigg) \\
&+ \alpha(1-\delta)1\{S(n,s',e') \geq 0\} p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'), S(n,s',e')\} \\
&- \alpha p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'), 0\}\Bigg]
\end{aligned} \tag{15}
$$

Conjecture that $S(n,s,e)$ is monotonically increasing in match quality n for a fixed state (s,e). This will be verified later (see Proposition 1 and Appendix A.3). Then the second term in the expectation becomes

$$\alpha(1-\delta)1\{S(n,s',e')\geq 0\}p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e'),S(n,s',e')\}$$
$$=\alpha(1-\delta)1\{S(n,s',e')\geq 0\}p(\theta(s,e_+))\left(\sum_{j=1}^{n}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}S(n,s',e')+\sum_{j=n+1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}S(j,s',e')\right)$$

and the joint surplus simplifies to

$$\begin{aligned}
S(n,s,e) = f(n,s) - h(s) + \beta\mathbf{E}_{s'|s}\Bigg[&(1-\delta)\max\{S(n,s',e'),0\}\\
&\times\Bigg(1-\alpha p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}-(1-\alpha)p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\\
&+\alpha p(\theta(s,e_+))\sum_{j=1}^{n}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\Bigg)\\
&+\alpha(1-\delta)1\{S(n,s',e')\geq 0\}p(\theta(s,e_+))\sum_{j=n+1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}S(j,s',e')\\
&-\alpha p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e'),0\}\Bigg]\\
= f(n,s) - h(s) + \beta\mathbf{E}_{s'|s}\Bigg[&(1-\delta)\max\{S(n,s',e'),0\}\left(1-p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right)\\
+\alpha(1-\delta)1\{S(n,s',e')\geq 0\}&p(\theta(s,e_+))\sum_{j=n+1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}S(j,s',e')\\
-\alpha p(\theta(s,e_+))&\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e'),0\}\Bigg]
\end{aligned}$$

The last expression is Equation (9) in the main text.

A.2 Derivation of wage

Taking the first-order conditions of Equation (10) yields

$$\begin{aligned}
0 &= (1-\alpha)J(n,s,e)^{-\alpha}\big(W(n,s,e)-U(s,e)\big)^{\alpha}\frac{\partial J(n,s,e)}{\partial W(n,s,e)}\\
&\quad+\alpha\big(W(n,s,e)-U(s,e)\big)^{\alpha-1}J(n,s,e)^{1-\alpha}\frac{\partial[W(n,s,e)-U(s,e)]}{\partial W(n,s,e)}\\
\iff 0 &= S^W(n,s,e)^\alpha J(n,s,e)^{1-\alpha}\left[\frac{1-\alpha}{J(n,s,e)}\frac{\partial J(n,s,e)}{\partial W(n,s,e)}+\frac{\alpha}{S^W(n,s,e)}\frac{\partial S^W(n,s,e)}{\partial W(n,s,e)}\right]\\
\iff 0 &= \frac{1-\alpha}{J(n,s,e)}\frac{\partial J(n,s,e)}{\partial W(n,s,e)}+\frac{\alpha}{S^W(n,s,e)}\frac{\partial S^W(n,s,e)}{\partial W(n,s,e)}
\end{aligned}$$

Using $\frac{\partial J(n,s,e)}{\partial w(n,s,e)} = -1$ and $\frac{\partial S^W(n,s,e)}{\partial w(n,s,e)} = 1$, we get Equation (11) in the main text. Next, note that

$$S(n,s,e) = W(n,s,e) - U(s,e) + J(n,s,e) = S^W(n,s,e) + \frac{1-\alpha}{\alpha} S^W(n,s,e) = \frac{1}{\alpha} S^W(n,s,e)$$

so $S^W(n,s,e) = \alpha S(n,s,e)$. Similarly, $J(n,s,e) = (1-\alpha)S(n,s,e)$. Using these, the identities $\max\{a-b,0\} = \max\{a,b\} - b$ and $1\{x \geq 0\} \times x = \max\{x,0\}$, and Proposition 1, the worker surplus simplifies to

$$\begin{aligned} S^W(n,s,e) = w(n,s,e) - h(s) + \beta \mathbf{E}_{s'|s} \Bigg[& (1-\delta)\left(1 - p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}}\right) \alpha \max\{S(n,s',e'),0\} \\ & - p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S^W(j,s',e'),0\} \\ & + \alpha(1-\delta) 1\{S(n,s',e') \geq 0\} p(\theta(s,e_+)) \sum_{j=n+1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} S(j,s',e') \Bigg] \end{aligned}$$

Plugging this and Equation (7) into the surplus splitting equation and rearranging terms yields the wage equation

$$\begin{aligned} w(n,s,e) = \alpha f(n,s) + (1-\alpha)h(s) + \alpha(1-\alpha)\beta \mathbf{E}_{s'|s} \Bigg[& p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'),0\} \\ & - (1-\delta) 1\{S(n,s',e') \geq 0\} p(\theta(s,e_+)) \sum_{j=n+1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} S(j,s',e') \Bigg] \end{aligned} \tag{16}$$

A.3 Proof of Proposition 1

Proof. Suppose not. Then $S(n+1,s,e) \le S(n,s,e)$, $\forall(s,e)$. Using Equation (15) and the identity $1\{x \ge 0\} \times x = \max\{x, 0\}$, note that $S(n+1,s,e) - S(n,s,e)$ can be written as

$$
\begin{aligned}
S(n+1,s,e) - S(n,s,e) = f(n+1,s) - f(n,s) + \beta \mathbf{E}_{s'|s} \Bigg[\\
&(1-\delta)1\{S(n+1,s',e') \ge 0\} S(n+1,s',e') \Bigg(1 - \alpha p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \\
&- (1-\alpha) p(\theta(s,e_+)) \frac{\sum_{k=n+2}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \Bigg) \\
&- (1-\delta)1\{S(n,s',e') \ge 0\} S(n,s',e') \Bigg(1 - \alpha p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \\
&- (1-\alpha) p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \Bigg) \\
&+ \alpha(1-\delta)1\{S(n+1,s',e') \ge 0\} p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'), S(n+1,s',e')\} \\
&- \alpha(1-\delta)1\{S(n,s',e') \ge 0\} p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'), S(n,s',e')\} \Bigg]
\end{aligned}
$$

where the last term inside the expectation cancels because it is independent of n. There are three cases to consider:

1. $0 \le S(n+1,s',e') \le S(n,s',e')$ which implies $1\{S(n,s',e') \ge 0\} = 1\{S(n+1,s',e') \ge 0\} = 1$
2. $S(n+1,s',e') \le 0 \le S(n,s',e')$ which implies $1\{S(n,s',e') \ge 0\} = 1$ and $1\{S(n+1,s',e') \ge 0\} = 0$
3. $S(n+1,s',e') \le S(n,s',e') \le 0$ which implies $1\{S(n,s',e') \ge 0\} = 1\{S(n+1,s',e') \ge 0\} = 0$

Note the case $S(n,s',e') \le 0 \le S(n+1,s',e')$ is not possible because it contradicts $S(n+1,s,e) \le S(n,s,e)$, $\forall(s,e)$.

Case 1. Since $1\{S(n,s',e') \geq 0\} = 1\{S(n+1,s',e') \geq 0\} = 1$, the above equation reduces to

$$
\begin{aligned}
S(n+1,s,e) - S(n,s,e) = f(n+1,s) - f(n,s) + \beta \mathbf{E}_{s'|s} \Bigg[& \\
& (1-\delta) S(n+1,s',e') \left(1 - \alpha p(\theta(s,e_+)) - (1-\alpha) p(\theta(s,e_+)) \frac{\sum_{k=n+2}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \right) \\
& - (1-\delta) S(n,s',e') \left(1 - \alpha p(\theta(s,e_+)) - (1-\alpha) p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \right) \\
& + \alpha(1-\delta) p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'), S(n+1,s',e')\} \\
& - \alpha(1-\delta) p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e'), S(n,s',e')\} \Bigg]
\end{aligned}
$$

Using the identity $\max\{a-b, 0\} = \max\{a,b\} - b$, the last two terms become

$$
\begin{aligned}
S(n+1,s,e) - S(n,s,e) = f(n+1,s) - f(n,s) + \beta \mathbf{E}_{s'|s} \Bigg[& \\
& (1-\delta) S(n+1,s',e') \left(1 - (1-\alpha) p(\theta(s,e_+)) \frac{\sum_{k=n+2}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \right) \\
& - (1-\delta) S(n,s',e') \left(1 - (1-\alpha) p(\theta(s,e_+)) \frac{\sum_{k=n+1}^{N} \frac{1}{k^\eta}}{\sum_{k=1}^{N} \frac{1}{k^\eta}} \right) \\
& + \alpha(1-\delta) p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e') - S(n+1,s',e'), 0\} \\
& - \alpha(1-\delta) p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e') - S(n,s',e'), 0\} \Bigg]
\end{aligned}
$$

Since $S(n+1,s,e) \leq S(n,s,e)$, the sum of the last two terms is weakly positive:

$$
\begin{aligned}
& \alpha(1-\delta) p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e') - S(n+1,s',e'), 0\} \\
& \qquad - \alpha(1-\delta) p(\theta(s,e_+)) \sum_{j=1}^{N} \frac{1}{j^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} \max\{S(j,s',e') - S(n,s',e'), 0\} \geq 0
\end{aligned}
$$

Finally, since the other two terms in the expectation are positive, we arrive at a contradiction.

Case 2. $1\{S(n,s',e') \geq 0\} = 1$ and $1\{S(n+1,s',e') \geq 0\} = 0$ so the above equation reduces to

$$S(n+1,s,e) - S(n,s,e) = f(n+1,s) - f(n,s) - \beta \mathbf{E}_{s'|s}\Bigg[$$
$$(1-\delta)S(n,s',e')\left(1 - \alpha p(\theta(s,e_+)) - (1-\alpha)p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right)$$
$$+ \alpha(1-\delta)p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e'),S(n,s',e')\}\Bigg]$$

Using the identity $\max\{a-b,0\} = \max\{a,b\} - b$, the last term becomes

$$S(n+1,s,e) - S(n,s,e) = f(n+1,s) - f(n,s) - \beta \mathbf{E}_{s'|s}\Bigg[$$
$$(1-\delta)S(n,s',e')\left(1 - \alpha p(\theta(s,e_+)) - (1-\alpha)p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right)$$
$$+ \alpha(1-\delta)p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e') - S(n,s',e'),0\}$$
$$+ \alpha(1-\delta)p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}S(n,s',e')\Bigg]$$
$$= f(n+1,s) - f(n,s) - \beta \mathbf{E}_{s'|s}\Bigg[(1-\delta)S(n,s',e')\left(1 - (1-\alpha)p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right)$$
$$+ \alpha(1-\delta)p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e') - S(n,s',e'),0\}\Bigg]$$

[to be completed] As in the first case, the RHS of this equation is positive, a contradiction.

Case 3. Since $1\{S(n,s',e') \geq 0\} = 1\{S(n+1,s',e') \geq 0\} = 0$, the above equation reduces to $S(n+1,s,e) - S(n,s,e) = f(n+1,s) - f(n,s) > 0$ because $f(\cdot,s)$ is strictly increasing, a contradiction. This completes the proof. $\square$

A.4 Adding a minimum wage

First, note that Equation (13) simplifies to

$$U(s,e;w_{\min}) = h(s) + \beta \mathbf{E}_{s'|s}\Bigg[U(s',e';w_{\min})$$
$$+ p(\theta(s,e_+))\alpha \max\{S(\hat{n}(s',e'),s',e';w_{\min}),0\}\sum_{j=1}^{\hat{n}(s',e')}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}$$
$$+ p(\theta(s,e_+))\alpha\sum_{j=\hat{n}(s',e')+1}^{N}\frac{1}{j^\eta\sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j,s',e';w_{\min}),0\}\Bigg],$$

where the joint value of a match is

$$
\begin{aligned}
S(n,s,e;w_{\min}) &= W(n,s,e;w_{\min}) - U(s,e;w_{\min}) + J(n,s,e;w_{\min}) \\
&= f(n,s) - h(s) \\
&\quad + \beta \mathbf{E}_{s'|s}\Bigg[(1-\delta)1\{S(n,s',e') \geq 0\}\Big(W(n,s',e') - U(s',e';w_{\min})\Big) \\
&\quad + (1-\delta)1\{S(n,s',e') \geq 0\}p(\theta(s,e_+))\sum_{j=1}^{N}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}}\max\{W(j,s',e') - W(n,s',e'),0\} \\
&\quad + (1-\delta)1\{S(n,s',e') \geq 0\}\left(1 - p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^{\eta}}}{\sum_{k=1}^{N}\frac{1}{k^{\eta}}}\right)J(n,s',e') \\
&\quad - p(\theta(s,e_+))\alpha\max\{S(\hat{n}(s',e'),s',e'),0\}\sum_{j=1}^{\hat{n}(s',e')}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}} \\
&\quad - p(\theta(s,e_+))\alpha\sum_{j=\hat{n}(s',e')+1}^{N}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}}\max\{S(j,s',e'),0\}\Bigg].
\end{aligned}
$$

Note here that all the terms in this equation are identical to what we had previously without the minimum wage policy except for the last two terms in the expectation. Hence, following the same steps as in Appendix A.1, the above simplifies to

$$
\begin{aligned}
S(n,s,e;w_{\min}) = f(n,s) - h(s) + \beta \mathbf{E}_{s'|s}\Bigg[&(1-\delta)\max\{S(n,s',e';w_{\min}),0\}\left(1 - p(\theta(s,e_+))\frac{\sum_{k=n+1}^{N}\frac{1}{k^{\eta}}}{\sum_{k=1}^{N}\frac{1}{k^{\eta}}}\right) \\
&+ \alpha(1-\delta)1\{S(n,s',e';w_{\min}) \geq 0\}p(\theta(s,e_+))\sum_{j=n+1}^{N}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}}S(j,s',e';w_{\min}) \\
&- p(\theta(s,e_+))\alpha\max\{S(\hat{n}(s',e'),s',e';w_{\min}),0\}\sum_{j=1}^{\hat{n}(s',e')}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}} \\
&- p(\theta(s,e_+))\alpha\sum_{j=\hat{n}(s',e')+1}^{N}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}}\max\{S(j,s',e';w_{\min}),0\}\Bigg],
\end{aligned}
$$

which is Equation (14) in the main text.

B Computational Appendix

This appendix details the solution methods used in this paper. Appendix B.1 describes the procedure to solve for the recursive stochastic equilibrium. Appendix B.2 describes the procedure to trace out the transition back to steady state following either an unexpected productivity shock or increase in the exogenous separation rate.

B.1 Solving the Stochastic Equilibrium

Solution Method. I solve the model as in Barlevy (2002) by approximating the total surplus and using the collocation method. The procedure is as follows:

1. Guess an initial distribution of workers across match types $e = \{e(n)\}_{n=1}^{N}$ and a sequence of shocks.

2. Solve for $S(n,s,e)$ for all (n,s) pairs by approximating it as a second-order polynomial in e with the collocation method (detailed below).

3. Invert the free entry condition to get market tightness.

4. Iterate forward using the law of motion to get e'.

5. Repeat this process in each period.

In practice, to reduce the computational burden of this solution method, I do not redo Step 2 every period. Once the collocation method has converged and I have the surplus as a function of the distribution of workers across match qualities and the state, I store it and use it for the following periods. Since I am solving the model $T = 3000$ times (the equivalent of 250 years), this greatly improves performance. After 600 periods (50 years), I re-approximate the surplus so as to attenuate the potential impact of initial conditions. I also discard these first 600 periods, which effectively act as a burn-in phase.

Collocation Method. The collocation method[20] in Step 2 imposes that the system of approximate value functions

$$\tilde{S}(n,s,e) = f(a^{(n,s)}, e)$$

holds with equality where $f(\cdot)$ is the right hand side of Equation (9) and $\tilde{S}(n,s,e)$ is a second order polynomial in e. It proceeds as follows:

1. Start by guessing a set of collocation points $(e_1^{(n,s)}, e_2^{(n,s)}, \ldots, e_N^{(n,s)})$ such that $\sum_n e_n^{(n,s)} \leq 1, \forall(n,s)$ pairs. Use these to generate the $N \cdot |s| \times \frac{(N+1)(N+2)}{2}$ matrix (which is held fixed throughout)

$$X = \begin{bmatrix} 1 & e_1^{(n,s)} \times e_1^{(n,s)} & e_1^{(n,s)} \times e_2^{(n,s)} & \ldots e_1^{(n,s)} \times e_N^{(n,s)} & e_2^{(n,s)} \times e_2^{(n,s)} & \ldots e_2^{(n,s)} \times e_N^{(n,s)} & \ldots & e_N^{(n,s)} \times e_N^{(n,s)} \end{bmatrix}_{\forall(n,s)}$$

   where $|s|$ denotes the number of states (under this specification, $|s| = 5$ and $N = 10$). Let $X^{(n,s)}$ denote the row corresponding to the (n,s) pair.

2. For each (n,s) pair, guess the $\frac{(N+1)(N+2)}{2} \times 1$ vector of coefficients $a_{\text{old}}^{(n,s)}$.

3. For all (n,s) pairs, approximate the surplus by

$$\tilde{S}(n,s,e) = a_0^{(n,s)} + \sum_{n=1}^{N} a_n^{(n,s)} e(n) + \sum_{k=1}^{N} \sum_{j=k}^{N} a_{j,k}^{(n,s)} e(j)e(k)$$

   Note that there are $1 + N + \frac{N(N+1)}{2} = \frac{(N+1)(N+2)}{2}$ coefficients for each (n,s) pair.

4. Compute the RHS using the approximated surplus and the market tightness from inverting the free entry condition. This yields, for each (n,s) pair, $f(a^{(n,s)}, e)$ (a scalar).

[20] see, e.g., Judd (1996) for a clear overview of the method.

5. We now have the following system of equations

$$X^{(n,s)} a_{\text{new}}^{(n,s)} = f(a_{\text{old}}^{(n,s)}, e)$$

So update a using

$$a_{\text{new}}^{(n,s)} = \left(X^{(n,s)\prime} X^{(n,s)}\right)^{-1} X^{(n,s)\prime} f(a_{\text{old}}^{(n,s)}, e)$$

6. Repeat until convergence, i.e. $a_{\text{new}}^{(n,s)} = a_{\text{old}}^{(n,s)}$ for all (n,s) pairs.

Implementation details. As previously stated, the model is solved and simulated for 250 years where the first 50 years are used as burn-in. I draw the collocation points and the initial guess for the distribution e_0 from a uniform distribution on the unit interval and normalize to ensure the sum is less than 1. I draw the guess of coefficients a from a Normal(1,0) distribution.

B.2 Impulse Response

I first describe how to solve for a steady state equilibrium, which is needed for the impulse response exercise. I then outline how the impulse responses are computed.

Steady State. First note that, in steady state and for a given productivity level, Equations (4), (8) and (9) reduce to

$$\begin{aligned} e(n) = {} & \left(1 - p(\theta)\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right)(1-\delta)1\{S(n) \geq 0\}e(n) \\ & + \frac{1}{n^\eta \sum_{k=1}^{N}\frac{1}{k^\eta}} p(\theta)\left(1 - \sum_{k=n}^{N}(1-\delta)1\{S(k) \geq 0\}e(k)\right) \end{aligned} \tag{17}$$

$$\kappa = q(\theta)(1-\alpha)\sum_{n=1}^{N}\frac{1}{n^\eta \sum_{k=1}^{N}\frac{1}{k^\eta}}\left(u_+ + \sum_{l=1}^{n-1} e_+(l)\right)\max\{S(n), 0\} \tag{18}$$

$$\begin{aligned} S(n) = f(n) - h + \beta\Bigg(& (1-\delta)\max\{S(n),0\}\left(1 - p(\theta)\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right) \\ & + \alpha(1-\delta)1\{S(n) \geq 0\}p(\theta)\sum_{j=n+1}^{N}\frac{1}{j^\eta \sum_{k=1}^{N}\frac{1}{k^\eta}} S(j) \\ & - \alpha p(\theta)\sum_{j=1}^{N}\frac{1}{j^\eta \sum_{k=1}^{N}\frac{1}{k^\eta}}\max\{S(j), 0\}\Bigg) \end{aligned} \tag{19}$$

Rearranging Equation (17), we get

$$e(n) = \frac{\frac{1}{n^\eta \sum_{k=1}^{N}\frac{1}{k^\eta}} p(\theta)\left(1 - \sum_{k=n+1}^{N}(1-\delta)1\{S(k) \geq 0\}e(k)\right)}{1 - \left(1 - p(\theta)\frac{\sum_{k=n+1}^{N}\frac{1}{k^\eta}}{\sum_{k=1}^{N}\frac{1}{k^\eta}}\right)(1-\delta)1\{S(n) \geq 0\} + \frac{1}{n^\eta \sum_{k=1}^{N}\frac{1}{k^\eta}} p(\theta)(1-\delta)1\{S(n) \geq 0\}}$$

and, in particular, for $n = N$ we have

$$e(N) = \frac{\frac{1}{N^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} p(\theta)}{1 - (1-\delta)1\{S(N) \geq 0\} + \frac{1}{N^\eta \sum_{k=1}^{N} \frac{1}{k^\eta}} p(\theta)(1-\delta)1\{S(N) \geq 0\}}.$$

This does not depend on the number of workers in all the other matches and so, given values $\{S(n)\}_{\forall n}$ and a market tightness θ, the distribution $\{e(n)\}_{\forall n}$ can be computed by iterating backwards, i.e. by first calculating $e(N)$, then $e(N-1)$, then $e(N-2)$, and so on.

I use a simple shooting method to solve for the steady state equilibrium. First, guess a value of θ. Then iterate on Equation (19) until convergence to get $\{S(n)\}_{\forall n}$. As an initial guess for the surplus, I use the present value of output less home production discounted by $(1-\delta)$, $S(n) = \frac{1}{\delta}(f(n) - h)$, $\forall n$. Next, solve for $\{e(n)\}_{\forall n}$ as described above. Finally, update the guess of θ by inverting Equation (18). Repeat until θ has converged.

Note that the steady state has the following intuitive two properties. First, steady state vacancy posting (or market tightness) is increasing in the aggregate state. Second, steady state unemployment is decreasing in the aggregate state.

Impulse Response. I set $T_P = 36$ so that the transition back to steady state takes 3 years. The impulse responses eare computed as follows:

1. Start with the economy in steady state at $t = -1$.
2. In period $t = 0$, the unexpected shock occurs.
3. Guess a path of vacancy creation $\{\theta_t\}_{t=0}^{T_P}$ (since $\theta_{-1}, \theta_{T_P+1}$ are known).
4. Iterate backwards to get the surplus $\{\{S(n, s_t, e_t)\}_{t=T_P}^{0}\}_{\forall n}$ (again, the surplus in the first and last period is known).
5. Iterate forward to get the distribution of workers across match type $\{e_{t+1}(n)\}_{t=0}^{T_P}$ (since e_0 is inherited from $t = -1$ and so is known).
6. Update the guess of $\{\theta_t\}_{t=0}^{T_P}$.
7. Repeat until the path for vacancies has converged.

B.3 Steady State with Minimum Wage

In steady state, the surplus under a binding minimum wage for a given productivity level is

$$\begin{aligned} S(n;w_{\min}) = f(n) - h + \beta\Bigg[&(1-\delta)\max\{S(n;w_{\min}),0\}\left(1-p(\theta)\frac{\sum_{k=n+1}^{N}\frac{1}{k^{\eta}}}{\sum_{k=1}^{N}\frac{1}{k^{\eta}}}\right) \\ &+\alpha(1-\delta)1\{S(n;w_{\min})\geq 0\}p(\theta)\sum_{j=n+1}^{N}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}}S(j;w_{\min}) \\ &-p(\theta)\alpha\max\{S(\hat{n};w_{\min}),0\}\sum_{j=1}^{\hat{n}(s',e')}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}} \\ &-p(\theta)\alpha\sum_{j=\hat{n}(s',e')+1}^{N}\frac{1}{j^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}}\max\{S(j;w_{\min}),0\}\Bigg]. \end{aligned} \tag{20}$$

The free entry condition and the law of motion are unaffected by the addition of the binding minimum and are given by Equations (17) and (18). To solve for the steady state equilibrium with a minimum wage, I first solve the unconstrained problem as described in Appendix B.2. I then compute $\hat{n} := \max\{n : w_{\min} \geq w(n)\}$. Finally, I redo the procedure described in Appendix B.2 using Equation (20) instead of Equation (19).

C Estimation Appendix

C.1 Model simulated moments

I compute the simulated moments to match how the moments are calculated in the data. The following time series are computed, for a given set of parameters, and used to compute the moments:

1. The unemployment to employment transition rate:

$$\mathrm{UE}_t = u_{t+}p(\theta(s_t,e_{t+}))\sum_{n=1}^{N}\frac{1}{n^{\eta}\sum_{k=1}^{N}\frac{1}{k^{\eta}}}1\{S(n,s_t,e_t)\geq 0\}/u_{t+}$$

2. The employment to unemployment transition rate:

$$\mathrm{EU}_t = \frac{\sum_{n=1}^{N}e_t(n)\Big(1\{S(n,s_t,e_t)<0\}+\delta 1\{S(n,s_t,e_t)\geq 0\}\Big)}{\sum_{n=1}^{N}e_t(n)}$$

3. The employer to employer transition rate:

$$\mathrm{EE}_t = \frac{p(\theta(s_t,e_{t+}))\sum_{n=1}^{N}e_{t+}(n)\frac{\sum_{k=n+1}^{N}\frac{1}{k^{\eta}}}{\sum_{k=1}^{N}\frac{1}{k^{\eta}}}}{\sum_{n=1}^{N}e_{t+}(n)} = \frac{p(\theta(s_t,e_{t+}))\sum_{n=1}^{N-1}e_{t+}(n)\frac{\sum_{k=n+1}^{N}\frac{1}{k^{\eta}}}{\sum_{k=1}^{N}\frac{1}{k^{\eta}}}}{\sum_{n=1}^{N}e_{t+}(n)}$$

since when $n = N$, the numerator is an empty sum.

4. The fraction of unemployed workers at period t who have been unemployed for more than k periods:

$$u_t^k = u_{t-k} \prod_{j=0}^{k-1} \left(1 - \sum_{n=1}^{N} \frac{1}{n^\eta \sum_{k=1}^N \frac{1}{k^\eta}} p(\theta(s_{t-k+j}, e_{t-k+j+})) 1\{S(n, s_{t-k+j}, e_{t-k+j}) \geq 0\}\right) / u_t$$

5. The mean wage:

$$w_t = \sum_{n=1}^{N} \frac{e_t(n)}{\sum_{k=1}^N e_t(k)} w(n, s_t, e_t)$$

6. The average starting wage out of unemployment:

$$\overline{w}_t^{UE} = \frac{u_{t+} p(\theta(s_t, e_{t+})) \sum_{n=1}^N \frac{1}{n^\eta \sum_{k=1}^N \frac{1}{k^\eta}} w(n, s_t, e_t)}{u_{t+} p(\theta(s_t, e_{t+}))} = \sum_{n=1}^{N} \frac{1}{n^\eta \sum_{k=1}^N \frac{1}{k^\eta}} w(n, s_t, e_t)$$

7. The average starting wage following an EE transition:

$$\begin{aligned}\overline{w}_t^{EE} &= \frac{\sum_{n=1}^N e_{t+}(n) p(\theta(s_t, e_{t+})) \frac{1}{n^\eta \sum_{k=1}^N \frac{1}{k^\eta}} \sum_{j=n+1}^N w(j, s_t, e_t)}{\sum_{n=1}^N e_{t+}(n) p(\theta(s_t, e_{t+})) \frac{\sum_{k=n+1}^N \frac{1}{k^\eta}}{\sum_{k=1}^N \frac{1}{k^\eta}}} \\ &= \frac{\sum_{n=1}^N e_{t+}(n) \frac{1}{n^\eta \sum_{k=1}^N \frac{1}{k^\eta}} \sum_{j=n+1}^N w(j, s_t, e_t)}{\sum_{n=1}^N e_{t+}(n) \frac{\sum_{k=n+1}^N \frac{1}{k^\eta}}{\sum_{k=1}^N \frac{1}{k^\eta}}}\end{aligned}$$

C.2 MCMC algorithm

Baragatti, Grimaud, and Pommeret (2013) build on Chernozhukov and Hong (2003) to address two issues. First, in some cases, the sampler does not explore the full parameter space. In particular, the tails can be left unexplored. Second, the samples from one iteration to the next can be very correlated. The algorithm proposed by Baragatti, Grimaud, and Pommeret (2013) to alleviate these concerns is as follows. Let N_c denote the number of chains. The set of parameters is $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_{N_c})$ with associated datasets $z = (z_1, z_2, \ldots, z_{N_c})$. The observed data is x. The state associated with chain i is $\{z_i, \varphi_i\}$. The algorithm to obtain a Markov chain of length n_c proceeds as follows:

1. Choose temperatures $T_1 = 1 < T_2 < T_3 < \cdots < T_{N_c}$ and tolerance levels $\epsilon_1 < \epsilon_2 < \cdots < \epsilon_{N_c}$. Chains of higher order are associated with high tolerance levels and higher temperatures so they are able to move more freely in the parameter space. Lower order chains give more precise approximations of the target posterior.

2. $t = 1$. Start with an initial value for the set of parameters for each chain, $\{\varphi_i^{(0)}, z_i^{(0)}\}_{i=1}^{N_c}$. In practice, I choose a different initial value for each education group.

3. For $t \in \{2, \ldots, n_c\}$, execute the following two types of moves:

   (a) Local moves. For $i \in \{1, 2, \ldots, N_c\}$:

i. Generate φ_i' from the transition kernel $q_i(\cdot|\varphi_i^{(t-1)})$ whose variance is an increasing function of the temperature T_i.

ii. Generate z_i' from φ_i' (i.e. simulate model and compute moments). Set $(\varphi_i^{(t)}, z_i^{(t)}) = (\varphi_i', z_i')$ with probability

$$\min\left\{1, \frac{\pi(\varphi_i')q_i(\varphi_i^{(t-1)}|\varphi_i')}{\pi(\varphi_i^{(t-1)})q_i(\varphi_i'|\varphi_i^{(t-1)})}\right\}1\{\rho(S(z_i'), S(x)) < \epsilon_i\}(z_i')$$

where ρ is a distance function and S is a statistic.

(b) Exchange moves. Since lower order chains may have some difficulties exiting local modes, N_c exchanges are proposed at each iteration. N_c pairs of chains (i, j) with $i < j$ are chosen uniformly from all possible pairs with replacement. The states of i, j are exchanged if

$$\rho(S(z_j), S(x)) < \epsilon_i$$

The parameter estimates are set to the mean of the coldest chain:

$$\hat{\varphi}^{MCMC} = \frac{1}{n_c}\sum_{i=1}^{n_c} \varphi_1^i$$

Implementation Details. I set the number of chains to $N_c = 25$ and the number of iterations to $n_c = 3000$. So the model is solved and simulated 75,000 times in total. The largest temperature is set to 2.

C.3 Untargeted moments

In Table 9, I present the model fit to a set of moments which were not targeted in estimation. The model does a good job at matching the volatility of the transition rates, but is very far from matching the wage dispersion in the data, as measured by the wage P90/P10.

Table 9: Untargeted moments

	No HS		HS		Some college		Associates		BA		Masters		Prof./PhD	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
sd(EU)	.006	.000	.003	.000	.003	.000	.002	.000	.002	.000	.002	.000	.002	.000
sd(UE)	.045	.059	.058	.074	.069	.083	.072	.082	.067	.091	.085	.066	.117	.107
$E[\overline{w}_t]$	1982	1934	3028	2316	3119	2595	3692	3427	5207	7102	6194	8989	7641	7120
$E[\overline{w}_t^{P9010}]$	6.83	1.79	5.16	1.69	7.19	1.61	5.08	1.58	5.79	1.45	5.19	1.49	4.86	1.38
sd($\overline{w}_t^{P9010}$)	.721	.054	.286	.066	.652	.065	.408	.060	.430	.051	.456	.044	.608	.049
sd($\overline{w}_t^{EE}/\overline{w}_t$)	.107	.023	.058	.029	.081	.027	.021	.029	.067	.025	.105	.020	.165	.034
sd(u_t^3)	.091	.073	.106	.084	.144	.091	.098	.084	.106	.069	.117	.047	.122	.089
$E[u_t^5]$	.340	.167	.361	.122	.362	.133	.311	.108	.343	.039	.317	.036	.372	.055

Note: To compute the time series and corresponding moments, the model is simulated for 3,000 periods (250 years) where the first 600 periods (50 years) are burn-in. To alleviate the effect of initial conditions, I do this 500 times and report the mean of each moment.