Dynamic Monopsony and Human Capital

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This version: November 15, 2023

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Abstract

A number of influential papers study monopsony power in static models. Among the papers that model dynamics with a finite number of firms, none model the process of human capital accumulation by workers. In this paper, I show that this has important implications for the measurement and welfare consequences of monopsony power. How large are properly measured markdowns? And what are the welfare gains of implementing competitive allocations once we have accounted for human capital accumulation? To answer these questions, I introduce a novel model of dynamic monopsony in which a large non-atomistic firm competes with a finite number of homogeneous firms for workers who learn on-the-job. The markdown has an additional dynamic term reflecting expected future changes in worker human capital. I estimate the model using rich matched employee-employer administrative data from France and find that the welfare gains from forcing firms to offer workers their marginal product are large. Moreover, the welfare losses are underestimated by 81% when ignoring human capital accumulation.

JEL codes: C45, D43, E24, J24, J42, L13.

Keywords: Dynamic Monopsony, Human Capital, Deep Learning.

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Introduction

Since the seminal work of Robinson (1932), a number of influential papers have attempted to measure monopsony power in static models.[1] Among the papers with dynamic oligopsony,[2] none considers the process of human capital accumulation by workers. However, there are two well-understood features of labor markets that, put together, give good reason to believe that monopsony should be studied in tandem with human capital accumulation: 1. monopsonists restrict employment by lowering wages below the marginal product of labor, and 2. employment is tightly linked to human capital accumulation through on-the-job learning.

In this paper, I show that these omissions have important implications for the measurement of monopsony power and its welfare costs. By ignoring human capital, existing work attributes wage markdowns, the gap between the wage and the marginal product of labor, to monopsony. What if workers are simply being paid less than their marginal product because of backloading coming from dynamic human capital accumulation? Or, in contrast, what if this provides a counterbalancing force to the standard *static* incentives to price below the marginal product because monopsony power induces the firm to care about the human capital accumulation of workers who are not at the firm? When allowing for dynamic human capital accumulation, the welfare implications of monopsony power become theoretically ambiguous and must be assessed using a quantitative model. What magnitude are properly measured markdowns? And what are the welfare gains of implementing competitive allocations once we have properly accounted for human capital accumulation?

I answer these questions by studying a dynamic model of imperfect competition in labor markets which I then quantify using French administrative data. I make five contributions. First, I document novel features of French local labor markets. I find that most markets are concentrated and, in particular, are composed of one large leading firm which is significantly larger than the other firms. Second, informed by this empirical evidence, I propose a model of a labor market[3] in which one large non-atomistic firm competes for workers with a finite number of homogeneous firms. Third, I modify tools from the machine learning literature on deep reinforcement learning to solve an otherwise intractable model. Fourth, I characterize the efficient allocation and show that the wage markdown, which now has a dynamic component reflecting human capital accumulation, no longer measures the gap to the efficient outcome. Finally, I use the estimated model to quantify the welfare losses from monopsony

[1] Static theories of monopsony include Manning (2011), Card, Cardoso, Heining, and Kline (2018), Dube, Giuliano, and Leonard (2019), MacKenzie (2019), Lamadon, Mogstad, and Setzler (2022), Berger, Herkenhoff, and Mongey (2022), and Yeh, Macaluso, and Hershbein (2022).

[2] Jarosch et al. (2023), Berger, Herkenhoff, Kostøl, and Mongey (2023), and Gottfries and Jarosch (2023).

[3] Throughout the paper, I define a local labor market to be the combination of a 4-digit occupation code and a commuting zone.

in French labor markets and to decompose what fraction of these losses can be attributed to the dynamics from human capital accumulation.

Measurement. I use rich employee-employer administrative data from France on the near-universe of workers to document several salient features of local labor markets. Defining a market as the intersection of a 4-digit occupation code and a commuting zone, I find that the majority of markets in France are very concentrated. Moreover, these markets employ a significant portion of the labor force. I also document that most markets are composed of one large *leading* firm, defined in terms of its employment share, which is significantly larger than all the other firms in the market. Finally, I examine the competitive structure of markets and find suggestive evidence that the leader does not strategically compete with the other firms.

Theory. Motivated by this empirical evidence, I propose a novel theory of imperfect competition in the labor market. The key departure relative to existing models of monopsony is that both the worker and firm problems are dynamic because of human capital accumulation. I model segmented markets composed of a non-atomistic *dominant* firm and a finite number of homogeneous *peripheral* firms. Workers are heterogeneous in their general human capital which they accumulate on-the-job: human capital appreciates when employed and depreciates when non-employed. Firms compete for workers by posting a menu of wages for each human capital level. There are four sources of monopsony power: 1. workers have idiosyncratic preferences for the different firms (and non-employment); 2. workers derive utility from firm-specific non-wage amenities; 3. workers are subject to switching costs whenever they transition from one firm to another or out of non-employment; and 4. the market is granular, i.e. there are a finite number of firms.

I assume the peripheral firms behave as if they are atomistic, effectively making them *static* monopsonists. They can exploit the market power due to the four sources mentioned above but do not internalize the dynamic effects when setting wages. In contrast, the dominant firm is a *dynamic* monopsonist. In addition to internalizing the effect its choice of wages has on its own labor supply curve, it also internalizes its effect on its future workforce and employment at the rival firms and non-employment. This offsets the standard incentive monopsonistic firms have to price below the marginal product of labor and introduces a tradeoff between current and future profits. Decreasing wages increases revenue today, but the future distribution of workers is less skilled. This is because some of the workers who would otherwise have been employed become non-employed and thus see their skills decay. In addition to the standard static term, the wage markdown of the dominant firm thus has a dynamic component reflecting expected changes in human capital.

In static models, the markdown, defined as the gap between the wage and the marginal product of labor, is equal the Lerner index of the firm's labor supply elasticity. The markdowns in this setting are still equal to the Lerner index, but for the dominant firm, the markdown is dynamic. It contains an additional term that reflects expected future changes in human capital. Moreover, in static models, the markdown also measures the distance between the wages offered and the wages that decentralize the efficient outcome and in that sense, is a sufficient statistic for the welfare losses due to monopsony.[4] To understand if this still holds with human capital accumulation, I then study the problem of a social planner who chooses the allocation of workers across firms and non-employment to maximize market-wide welfare. I show that setting both the dominant and peripheral wages equal to the *static* marginal product of labor decentralizes the efficient allocation. Since the marginal product of labor includes a dynamic term reflecting expected changes in future human capital, the dominant firm's markdown is no longer a sufficient statistic to measure the welfare losses.

Results. I estimate the model separately for two different representative markets, each containing half of the workers in France. Markets are split according to the employment share of the leader: the *bottom* and *top* representative markets correspond to markets where the leader's share is below and above the median, respectively. The most pervasive characteristic of these two resulting markets is that they are composed of a large leading firm, which employs considerably more workers than the other firms. The bottom representative is less concentrated, but the leading firm still employs roughly 5% of workers in the market, about 10 times more than the other firms. In contrast, in the top representative market, the leader employs 36% of workers, which is 14 times more than other firms.

I use the estimated model to quantify the welfare losses from monopsony in these two markets. I find that the peripheral firms pay workers 94 and 62 percent of their *static* marginal product, on average, in the bottom and top representative markets, respectively. In contrast, workers are not as sensitive to wage changes at the dominant firm which allows it to have a slightly lower markdown than the peripheral firms. The dominant firm pays workers 92 and 55 percent of their *dynamic* marginal product, on average, in the bottom and top representative markets, respectively. However, the dominant firm's markdown is not equal to the gap between the wages offered and the wages that decentralize the efficient allocation. I find that the markdown underestimates this gap: wages are only 35 and 18 percent of the efficient wages for the bottom and top representative markets, respectively.

Next, I study the effects of forcing *all* firms to price competitively. This entails raising

[4] Note the markdown does not capture the actual losses, but the distance to the efficient allocation.

wages by several orders of magnitude to match the wages that decentralize the efficient outcome. Since the dominant firm has more market power, leveling the playing field implies larger wage increases at the dominant firm relative to the peripheral firms. Moreover, output per worker is higher at the dominant firm which also benefits from a productivity advantage. I find that welfare is 59% and 68% smaller relative to the planner's solution in the bottom and top markets, respectively. These losses are in part driven by large output differences as employment at the dominant firm is much lower than it would be if it offered higher wages. The efficient allocation not only involves reallocation across firm types, i.e. from the peripheral firms to the more productive dominant firm, but also an employment effect. Therefore, there are also significant human capital gains to forcing firms to price competitively: I find that the aggregate distribution of human capital shifts to the right (less low skill and more high skill workers) in the efficient outcome.

Finally, to illustrate the importance of accounting for human capital accumulation, I do a decomposition exercise where I shut off the dynamics in the model. I then solve for the efficient outcome and compute the welfare losses relative to the decentralized equilibrium. These are considerably smaller than the ones estimated in the full model: the welfare losses are underestimated by 81 percent, on average, when ignoring the human capital accumulation of workers.

Literature. This paper contributes to several literatures. The most closely related work is a growing literature that models imperfect competition in the labor market (Burdett and Mortensen 1998; Manning 2003; Card, Cardoso, Heining, and Kline 2018; Kroft, Luo, Mogstad, and Setzler 2021; Lamadon, Mogstad, and Setzler 2022; Berger, Herkenhoff, and Mongey 2022; Engbom and Moser 2022; Gottfries and Jarosch 2023; and Jarosch, Nimczik, and Sorkin 2023, amongst many others). I depart from these benchmark frameworks in two main ways. First, I add worker heterogeneity in the form of endogenous human capital accumulation. This allows me to not only quantify inefficiencies due to market power *across* firms, but also *within* a firm. Second, I add dynamics: workers and firms are forward-looking and make their choices by fully internalizing their dynamic effects.[5] This addition significantly alters the measurement and welfare implications of monopsony. The dominant firm's markdown is dynamic with a new term reflecting expected changes in human capital and no longer fully measures the inefficiency.

[5] As mentioned above, there are several other papers that model dynamic monopsony. The seminal work of Burdett and Mortensen (1998) considers the limiting case $r/\lambda_0 \to 0$ where r is the discount rate shared by workers and firms and λ_0 is the Poisson arrival rate of a job offer for unemployed workers. As explained in their footnote 4, this implies firms post wages to maximize steady-state profits, hence effectively shutting off dynamics.

In all frictional models of the labor market, firms have monopsony power. Hence, search and matching models in which workers differ in their human capital (e.g. Barlevy 2008 and Bagger, Fontaine, Postel-Vinay, and Robin 2014, Lentz and Roys 2015, Flinn, Gemici, and Laufer 2017, amongst many others) can also be used to study the interaction of monopsony and on-the-job learning. In contrast, my model has four distinct sources of firms' market power: moving costs (isomorphic to search frictions), preference heterogeneity, amenities, and a finite number of employers. This granularity provides a more transparent mapping to the data and is better suited for analyzing policies. In this regard, this work strongly complements Gottfries and Jarosch (2023) who also argue in favor of explicitly modeling granular markets in the context of studying non-compete bans.

The methodology developed extensively draws upon Ericson and Pakes (1995)-style dynamic oligopoly models[6] and the large literature on dynamic discrete choice (e.g. the seminal works of Miller 1984; Wolpin 1984; Rust 1987; and Hotz and Miller 1993, amongst many others[7]). The model can be recast as a dynamic model of multi-product firms where, instead of competing in the product market for consumer demand (*oligopoly*), firms are competing in the input market for workers (*oligopsony*). An important departure is that, in these models, the *demand* for products is static, whereas here, the *supply* of workers is dynamic because of human capital accumulation. However, by assuming the peripheral firms are atomistic, the strategic interactions in my model are trivial as the peripheral firms offer workers a constant *static* markdown. Another contribution is to make progress on the tractability of this class of models. I rely on methods developed in the machine learning literature on deep reinforcement learning to solve for the decentralized equilibrium. In particular, I use neural networks to approximate key model objects (values, policies, and laws of motion) and modify the deep deterministic policy gradient algorithm proposed by Silver et al. (2014) and Lillicrap et al. (2015) to solve the firm problem. Importantly, this allows me to bypass the curse of dimensionality present because the dominant firm is non-atomistic and has a high dimensional aggregate distribution as a state variable. Thus, this paper also relates to a growing line of research using neural networks to solve general equilibrium models (Maliar, Maliar, and Winant 2021; Azinovic, Gaegauf, and Scheidegger 2022; Batzer 2022; Han, Yang, and E 2022; Kahou, Fernández-Villaverde, Perla, and Sood 2022; Kase, Melosi, and Rottner 2022; Duarte, Duarte, and Silva 2023; Fernández-Villaverde, Hurtado, and Nuño 2023). Finally, in estimation, I build on the literature on estimating models of dynamic discrete choice. I use a two-step identification procedure which allows me to first estimate the supply-side parameters from the conditional choice probabilities and the aggregate law of motion using

[6] See Doraszelski and Pakes (2007) for a great survey.

[7] See Aguirregabiria and Mira (2010) for a recent survey of this literature.

indirect inference and then estimate the remaining technology parameters by targeting the wages.

Outline. The rest of the paper is structured as follows. In Section 1, I describe the data and document several salient features of French labor markets. Section 2 introduces the model and Section 3 studies the planner's problem and characterizes the efficient allocation. In Section 4, I estimate the model and assess the model fit. Section 5 quantifies the human capital costs of monopsony. Finally, Section 6 concludes. Appendices A to D provide additional details on the data, model, omitted proofs, computation, and estimation.

1 Data and Descriptive Evidence

In this section, I provide a brief overview of the datasets used and document several salient features of French labor markets that will guide modeling decisions.

1.1 French administrative data

I use French administrative data commonly referred to as *Déclarations annuelles de données sociales* (DADS).[8] The DADS are mandatory forms that all businesses in France must submit annually. The French National Statistical Institute, *Institut national de la statistique et des études économiques* (INSEE), uses these forms to create matched employer-employee data allowing researchers to link workers and firms over time. I use two main datasets:

Short panel. The first dataset I use is the DADS-Postes, which covers the near-universe of jobs in France. This is a spell-based dataset, meaning there is one observation per individual-job-year.[9] Hence a worker with two jobs appears as two distinct observations. However, the worker IDs are reshuffled every two years, making it impossible to match workers across time for more than two periods. I use this to document descriptive evidence on French labor markets which I present in Section 1.2 and Section 1.3.

Long panel. The second dataset I use is the DADS-Panel. It contains the full employment history of all workers born in October of even years before 2002 and of every year from 2003 onwards. I treat it as a representative sample of the French labor force which can be used

[8] This translates to "Annual declaration of social data".

[9] For example, if a worker has two jobs in a given year, they appear as two separate observations with the same worker ID.

for longitudinal analysis.[10] Once a worker enters the dataset, all her subsequent employment spells are observable. I use this to measure human capital when I estimate the model.

Both sources are at the annual frequency and contain roughly the same worker-level variables.[11] These include detailed occupation codes, industry codes, zipcodes of residence and work, wages, hours, and demographics such as age and gender. See Appendix A.1 for a description of the primary variables used. However, the DADS has several drawbacks. The main disadvantage in the context of this study is that worker education is not observable. Instead, I use other worker characteristics to estimate unobserved human capital, which I describe in detail in Section 4.3. Another limitation is the absence of firm balance sheet data. While many firm and establishment level information such as wage bills and employment are available, data on revenue or input choices is not.[12] Finally, since workers only appear in the data when they are employed, we cannot distinguish between non-employment and unemployment. For this reason, I do not model unemployment and restrict workers to be either employed or non-employed.

Sample construction. I apply the same sample construction rules for both datasets. I focus on workers aged 16-65 who are employed in the private sector in metropolitan France. Furthermore, I exclude workers whose hourly gross wage falls outside three standard deviations of the mean. For the DADS-Postes, I use the repeated cross-section of 2014-2015 (henceforth DADS-Postes 2015). For the DADS-Panel, I go back to 2003,[13] the first year following a major change in occupation codes, up until 2016. An additional period after 2015 is needed to identify non-employed workers in 2015. In Table 1, I report basic summary statistics for both samples in 2015.

[10] There are birth month effects in labor markets as well as seasonality patterns to births that could in principle induce non-random selection into this subsample.

[11] Firm and establishment-level data are also available in the DADS-Entreprises and DADS-Etablissements datasets, respectively. I do not make use of these.

[12] Firm balance sheet information for French firms exists (the FARE/FICUS datasets), but due to legal restrictions is not available to researchers in North America.

[13] The DADS-Panel is also bigger after 2003. Before 2003, only workers born in October of even years are in the dataset, whereas after (and including) 2003, every worker born in October is included.

Table 1: Sample Summary Statistics

	Cross-section	Panel
Sample Size		
Workers	12,923,194	785,262
Firms	1,168,416	190,674
Year	2015	2015
Demographics		
Men (%)	56	56
Mean Age	40	39
Annual Earnings		
Mean	23,452	23,967
SD	12,636	25,567
p25	13,832	5,487
Median	23,140	19,894
p75	31,412	33,040

Note: This table reports summary statistics for the DADS-Postes 2015 and the DADS-Panel 2015. Both samples are constructed as described in Section 1.1. Wages are in 2015 euros.

1.2 Defining a labor market

Deciding how to set the boundaries of a labor market is a crucial choice that will affect both estimation and results. On the one hand, too narrow of a definition mechanically results in the creation of artificial concentration. It also risks separating workers who would be applying for the same jobs. On the other hand, if defined too broadly then it may lead to markets where firms do not overlap in their competition for workers. This risks pooling workers together that do not have the same jobs.

It is standard in the monopsony literature to define a labor market as some combination of geography, occupation, and/or industry. There are varying levels of granularity observable for each in the data: the whole of France, regions, départements, and commuting zones;[14] 1-,2-, and 4-digit occupation codes; and 3-digit industry codes. To the best of my knowledge, only two other studies use the same data and also need to define labor markets.[15] Azkarate-Askasua and Zerecero (2022) use the intersection between a commuting zone, a

[14] Before the 2016 reform, metropolitan France was split into 22 regions, 96 départements, and 305 commuting zones.

[15] Morlacco (2020) also uses French data, but instead uses trade and production data to study buyer power in input trade.

3-digit industry code, and a 2-digit occupation code and Arquié and Bertin (2022) use the intersection of a 4-digit occupation code and a commuting zone. I follow Arquié and Bertin (2022) and define a labor market in France as the intersection of a 4-digit occupation code and a commuting zone.

This decision is based on several factors. Using 1- or 2-digit codes for occupation results in very different occupations being grouped together. Table A.1 in Appendix A.3 illustrates occupation coding in the DADS for one of the 1-digit occupations, "Intermediate Professions", which includes the 2-digit occupation "Intermediate health and social work professions". Within this 2-digit occupation code, there are 21 different 4-digit occupations such as "nursing executives", "childcare workers", "midwives", "eyewear opticians", and "pharmacy assistants". Clearly, these are very different professions. Childcare workers and eyewear opticians are not applying for the same jobs. Table 2 reports self-flow rates and Table A.2 in Appendix A.3 reports additional statistics for both occupation codes. The self-flow rate along a dimension x (e.g. a region or a market) is the percent of workers who were labeled as x in $t-1$ and remain so in t. Although not a sufficient statistic, it is commonly reported to assess the accuracy of a market segmentation. Broadly speaking, a local labor market is the set of all jobs a worker would accept, therefore a helpful metric is how many workers remain in the same market over time. As expected, the self-flow rates are higher for the less granular occupation codes: 24% of workers switch 4-digit occupations from one year to the next, compared to 17% for 2-digit occupations. For geography, I follow the literature and use commuting zones. A commuting zone is a geographical unit defined by INSEE within which most workers both live and work.[16] A map of the 304 metropolitan commuting zones in France is relegated to Figure A.1 in Appendix A.4. From one year to the next, roughly 7.2% of workers work in a different commuting zone, compared to 6.7% for the département (96 in metropolitan France), and 2.9% for the region (13 in metropolitan France). Summary statistics for commuting zones can be found in Table A.3 in Appendix A.4. Finally, I do not additionally condition on the industry because that would result in the mechanic creation of concentration. Defining a market as the intersection of a 4-digit occupation code, a commuting zone, and a 3-digit industry code yields 1,287,951 distinct markets with a self-flow rate in 2015 of approximately 70%.

[16] I use the 2010 classification of commuting zones. Details on methodology can be found on INSEE's website.

Table 2: Self-Flow Rates

	Rate (%)
Region	97.12
Department	93.31
Commuting Zone (CZ)	92.81
3-digit ind	88.95
2-digit occ	83.05
4-digit occ	75.54
2-digit occ×CZ	79.43
4-digit occ×CZ	73.09
4-digit occ×CZ×3-digit ind	69.80

Note: Calculated in the DADS-Postes from 2014 to 2015.

There are 360 4-digit occupations and 305 commuting zones yielding 96,922 distinct labor markets with a self-flow rate of 73%. The majority of market outflows are due to occupation switching. Table 3 reports standard summary statistics on markets. The average market has 45 firms, 158 workers, and an employment Herfindahl-Hirschman Index (henceforth HHI) of 2,907. The employment HHI, the sum of the squared employment shares of firms in a market, is a standard measure of market concentration (see Appendix A.2 for more). A market with an employment HHI of 2,907 would be classified by the Horizontal Merger Guidelines (DOJ and FTC 2010) as highly concentrated. For example, a market with 4 firms, 3 of which employ 30% of workers corresponds to an employment HHI of 2,800. The average worker is in a market with 1,111 firms, 5,170 workers, and an employment HHI of 1,222. This is right below the 1,500 cutoffs the Horizontal Merger Guidelines (DOJ and FTC 2010) use to classify a market as moderately concentrated. For example, a market with 10 firms of equal size has an employment HHI of 1,000. Table A.4 in Appendix A.6 reports the same statistics when defining markets as the intersection of a 2-digit occupation code and a commuting zone.

Table 3: Market Summary Statistics

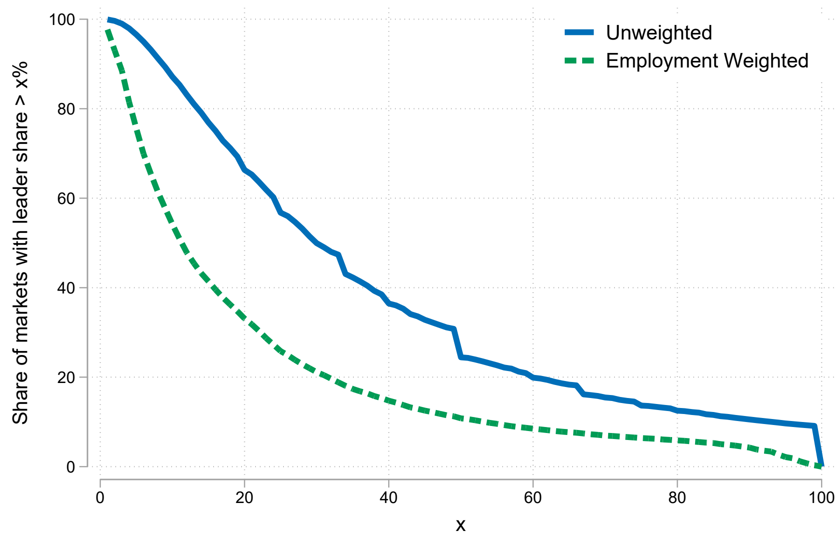
	Mean	SD	p25	Median	p75	p95
Number of Firms (unweighted)	45	232	4	11	33	162
Number of Firms (weighted)	1,111	2,920	49	167	625	6,925
Number of Workers (unweighted)	158	890	8	30	104	581
Number of Workers (weighted)	5,170	12,823	245	820	2,994	31,412
Employment HHI (unweighted)	2,907	2,939	734	1800	3,956	10,000
Employment HHI (weighted)	1,222	2,144	110	351	1,150	7,497
Number of Markets	96,922					

Note: Unweighted statistics computed with equal weights for each market. Weighted statistics use market employment as weights.

1.3 Market Structure

I now document several features of the competitive structure of labor markets in France which I will use to inform modeling assumptions in Section 2.

Most markets have a leader. I define the leading firm in a market as the firm with the highest employment share. Figure 1 looks at the distribution of the leader's employment share across markets. In 65% of markets (blue solid line), the largest firm has an employment share bigger than 20%, and 35% of workers in France (green dashed line) are employed in these markets. This distribution has a heavy right tail: in 20% of markets, the share is larger than 60%. However, only about 10% of workers are in these markets.



Note: The solid blue line plots, for each $x \in [0, 100]$, the percent of markets where the largest firm employs more than x% of workers. The green dashed line indicates what percent of workers are in such markets. Computed in the DADS-Postes 2015.

Figure 1: Prevalence of a leading firm

Although informative, Figure 1 does not capture the whole picture. It could be that the markets with the largest leaders are simply ones with a very small number of firms. As Table 3 shows, these markets also tend to employ fewer workers. I investigate this further by looking at the joint distribution of the market size (number of firms in a market) and a measure of how big the leader is relative to the other firms. In Figure 2, I report the employment-weighted heatmap between two variables: 1. the ratio of the leader's employment share to the mean employment share of the non-leader firms, and 2. the market size. Each number corresponds to the percentage of workers in France that are employed in these types of markets. The top left corner corresponds to the most concentrated markets: there are relatively few firms and the leader is significantly bigger than the other firms. Approximately 2% of workers are employed in these markets. Markets in the bottom left corner employ a larger fraction of workers. These are markets where there are fewer firms and the size of the leader relative to other firms is smaller. Nonetheless, these are still concentrated markets: consider for example the 1.97% of workers employed in markets where the leader is 4.8-7.3 times bigger than the other firms and there are 18-37 firms in total. Assuming the non-leader firms are of the same size and using the conservative bounds implies the leader and non-leader shares are approximately 12% and 2.5%, respectively. Lastly, perhaps the most striking feature of this heatmap is that conditional on the market having a very large number of firms, the gap between the leader and non-leader firms is also very large, i.e. there is no mass in the bottom right corner. See Figure A.2 in Appendix A.5 for the heatmap between the leader's employment share and the number of firms in the market.

Note: This plots the joint distribution of the ratio of the leader's employment share to the mean employment share of the non-leader firms and the market size (number of firms). Both variables are employment-weighted and split into deciles so that each row and column contains 10% of workers. For example, 10% of workers are in markets where there are between 1 and 18 firms.

Figure 2: Heatmap between Size of Leader and Number of Firms (Deciles)

The leader is much bigger than the other firms. As the y-axis of Figure 2 suggests, the vast majority of workers in France are in markets where the largest firm employs considerably more workers than the other firms in the market: 90% of workers are in markets where this ratio is larger than 4.8. In Table 4, I report summary statistics for several different measures to corroborate this finding. The average worker is in a market where the leader's share is 20%, which is six times larger than the employment share of the second largest firm and 127 times larger than the mean employment share of non-leader firms. These moments are driven by a heavy right tail: the median worker is in a market where the leader employs 11% of workers, which is 22 times more than the average non-leader firm and 50% more than the second biggest firm. The corresponding unweighted numbers are much larger, providing further evidence that the most concentrated markets do not employ the most workers. Table A.5 in Appendix A.6 reports the same statistics when defining markets as the intersection of a 2-digit occupation code and a commuting zone.

Table 4: Leader Summary Statistics

	Mean	SD	p25	Median	p75	p95
Leader Share (unweighted)	32	22	15	28	48	75
Leader Share (weighted)	20	22	5	11	25	73
Z-score of Leader Share (unweighted)	32	22	15	28	48	75
Z-score of Leader Share (weighted)	20	22	5	11	25	73
Leader Share / Second Share (unweighted)	3.1	8.0	1.3	1.8	2.6	8.0
Leader Share / Second Share (weighted)	6.0	34	1.2	1.5	2.5	12
Mean Non-Leader Share (unweighted)	7.0	7.2	2.0	4.6	9.5	23
Mean Non-Leader Share (weighted)	1.4	2.5	0.2	0.5	1.5	5.5
Leader Share / Mean Non-Leader Share (unweighted)	11	64	3.0	5.1	10	33
Leader Share / Mean Non-Leader Share (weighted)	127	911	10	22	55	1,250

Note: Unweighted statistics computed with equal weights for each market. Weighted statistics use market employment as weights. Markets with only one firm (leader's employment share = 100%) are excluded.

Strategic Interactions. Finally, I examine the extent to which the leader and non-leader firms compete. Absent firm balance sheet data, I look at changes in wages. I estimate the following market-level specification,

$$\Delta w_{i,t}^{N} = \beta_0 + \beta_1 \Delta w_{i,t}^{L} + \epsilon_{i,t} \tag{1}$$

where i indexes the market, t is a year, and $\Delta w_{i,t}^{j}$ is the mean hourly wage growth, in percents, for workers at firm type $j \in \{\text{Leader,Non-Leader}\}$ from $t-1$ to t. The leader in a market is defined as the firm with the largest employment share in $t-1$. Figure 3 plots changes in the mean non-leader hourly wage as a function of changes in the mean leader hourly wage. The dashed green line is the regression line. The slope is $\hat{\beta}_1 \approx 0.1$ (column (1) of Table 5): a 1% increase in mean wages offered by the dominant firm is associated with a 0.1% increase in mean wages offered by the non-leader firms. Given the response is biased upwards due to common market-level shocks faced by all firms, a 0.1% response is surprisingly low and suggests that the leader and non-leader firms do not strategically interact. In a fully competitive setting, the expected response would be 1-1, i.e. the 45-degree line in Figure 3. Weighting by employment (column (2)) and adding fixed effects for both occupation and commuting zone (column (3)) does not alter the result.

Figure 3: Wage Response of Non-Leader Firms

	Mean Non-Leader Wage		
	(1)	(2)	(3)
Mean Leader Wage	0.130***	0.126***	0.145***
	(0.005)	(0.051)	(0.045)
Fixed Effects	*N*	*N*	Y
Weighted	*N*	Y	Y
R^2	0.0135	0.0058	0.2676
N	93,183	93,183	93,183

Table 5: Estimates of Equation (1)

Note: Computed in DADS-Postes 2014-2015 for markets with more than one firm. **Figure.** This is a binned scatter plot with 100 bins: for each percentile along the x-axis, the mean value of the y-variable is plotted. The dashed green line is the regression line corresponding to column (1) in the table and the solid gray line is the 45-degree line. **Table.** I exclude markets for which wage growth falls outside 3 SDs of the mean. Fixed effects for 4-digit occupation and commuting zone. Standard errors are presented in parentheses.

1.4 Taking stock

Defining labor markets as the intersection of a 4-digit occupation code and a commuting zone, I find that many markets in France are concentrated. Moreover, these concentrated markets employ a significant fraction of workers. I also show that markets are composed of a large leading firm that is much bigger than other firms in the market. Finally, I find suggestive evidence that the leader does not compete with the non-leader firms. These features of the data highlight the need for a model in which the largest firm knows it is big and fully internalizes how it affects market-wide outcomes.

2 Model

In this section, I present a model of a monopsonistic labor market in which workers accumulate general human capital over time. The market is composed of one large non-atomistic firm that competes with a finite number of homogeneous firms for workers. Importantly, both workers and firms make dynamic choices. An accompanying table summarizing model notation can be found in Appendix B.1.

2.1 Setup

Environment. Time is discrete and the horizon is infinite, $t = 1, 2, \ldots$. There is no aggregate risk.

Agents. The economy is populated by a continuum of workers with measure 1. Workers are heterogeneous in general human capital h which takes values in the discrete set $\mathcal{H} := \{h_1, h_2, \ldots, h_H\}$ where $h_1 < h_2 < \cdots < h_H$ and $H \in \mathbb{N}$. Workers are either non-employed or employed at a firm.[17] At rate $\mu \in (0, 1)$, workers exogenously leave the labor force.[18] For each worker that exits, a new worker is born into non-employment at the bottom of the human capital ladder, i.e. with $h = h_1$. This birth-death process ensures the measure of workers in the economy remains constant and also dampens human capital accumulation which I describe below. There are two types of firms:

1. a finite number $M \in \mathbb{N}$ of homogeneous *peripheral* firms with common productivity p_P normalized to 1, and

2. a *dominant* firm with productivity $p_D \geq 1$.

A key model ingredient is that the dominant firm is non-atomistic, i.e. it internalizes the effect its choices have on the aggregate (market-wide) state. The set of peripheral firms is denoted $\mathcal{P} := \{y_1, y_2, \ldots, y_M\}$. I assume the peripheral firms behave as if they are atomistic: although there are finitely many of them, they do not internalize their effect on the aggregate distribution. This behavioral assumption is both motivated by the empirical evidence presented in Section 1 as well as tractability concerns which I discuss in Section 2.9. Note this is not as restrictive as assuming the peripheral firms behave competitively as they will still offer workers a wage below their marginal product. However, this wage (and markdown) is static and does not take into account any of the dynamics arising from the human capital accumulation of workers. I discuss this assumption more in Section 2.5 and Section 2.6 when I present the firm values.

Preferences. Both workers and firms are risk neutral and discount the future with common discount factor $\beta \in (0, 1)$. Firms compete for workers by posting wages for every human capital level to maximize the present discounted value of profits. I denote these choices $w_i := (w_i^1, \ldots, w_i^H)$ where $i \in \{D, P\}$ for *dominant* and *peripheral* indexes the firm type.

At each period t, a worker observes their state and chooses a destination $d_t \in \{D, y_1, \ldots, y_M, N\}$, where N is non-employment, to maximize the sum of present and expected future utilities.

[17] To be consistent with the data, I do not distinguish between non-employment and unemployment.
[18] This can be thought of as death or (exogenous) retirement.

As is standard in Markov decision models, the worker's optimal decision rule is the same in two different periods if the worker's states are equal. I therefore omit time subscripts going forward. In addition to their human capital h, workers are also heterogeneous in their time-varying preferences for firms and non-employment, denoted by $\epsilon := (\epsilon_D, \epsilon_{y_1}, \epsilon_{y_2}, \ldots, \epsilon_{y_M}, \epsilon_U)$. These idiosyncratic preferences reflect non-wage amenities, working conditions, and commuting costs, amongst many other things, and are unobservable to the researcher. I assume that ϵ are drawn *i.i.d* from a Type 1 Extreme Value distribution every period. For every alternative $d \in \{D, y_1, \ldots, y_M, N\}$, the CDF of ϵ_d is

$$G(\epsilon_d) = \exp(-\exp(\epsilon_d/\sigma)),$$

where I normalize the location to 0 and $\sigma > 0$ is the scale parameter.[19] In addition to the taste shocks, workers derive utility from non-wage amenities at both types of firms. I normalize the amenities at the peripheral firms to 0 and denote by $a_D := (a_D^1, a_D^2, \ldots, a_D^2)$ the vector of amenity values workers receive at the dominant firm. Finally, workers are subject to two moving costs: $\phi^{EE} > 0$ when they switch firms and $\phi^{NE} > 0$ when they move from non-employment to employment.[20] I assume utility is linear, time separable, and that the unobservable component ϵ_d enters additively into worker utility.

Technology. Firms produce a homogeneous final good with a price normalized to 1. The output produced by a firm with productivity $p_i \in \{p_D, 1\}$ and endogenous workforce $m_i := (m_i^1, m_i^2, \ldots, m_i^H)$ is

$$F_i(m_i) := p_i \sum_{h \in \mathcal{H}} m_i^h \times h \tag{2}$$

m_i^h is the number of workers with human capital h hired by firm type i. A non-employed worker of type h produces b^h, which can be thought of as home production.

Human Capital. A worker's human capital evolves according to their employment status. An employed worker with human capital h moves up one step in the human capital ladder with probability g_E^h:

$$h' = \begin{cases} h = h_k & \text{w/ prob.} \quad 1 - g_E^h \\ \min\{h_{k+1}, h_H\} & \text{w/ prob.} \quad g_E^h. \end{cases} \tag{3}$$

[19] The scale parameter governs the variance of the shocks $\pi^2\sigma^2/6$.

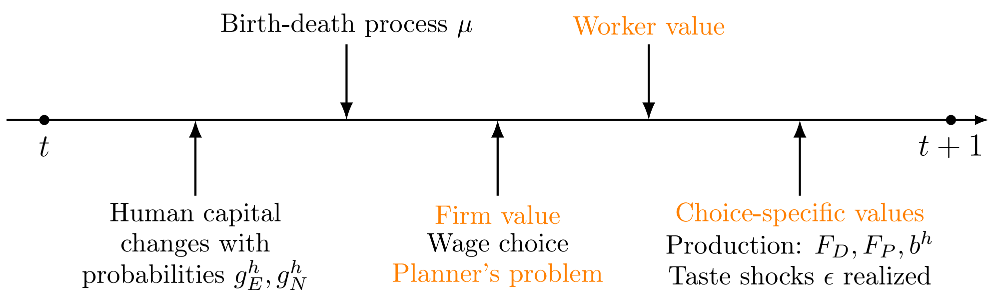
[20] If there were no cost to moving from N to E, then workers could go to non-employment for one period before going to employment to avoid the switching cost ϕ^{EE}.

Notice that there is no firm-specific learning rate: g_E^h is not indexed by the firm type $i \in \{D, P\}$. I follow Ljungqvist and Sargent (1998) and assume that human capital declines in non-employment. A non-employed worker can either move down one rung of the human capital ladder with probability g_N^h or stay at their current human capital level:

$$h' = \begin{cases} \max\{h_1, h_{k-1}\} & \text{w/ prob.} \quad g_N^h \\ h = h_k & \text{w/ prob.} \quad 1 - g_N^h. \end{cases} \tag{4}$$

In the value functions below, I will use $\mathbb{E}_{h'|h,d}$ to denote the human capital dynamics of a worker with current human capital h at destination d.

2.2 Timing and Labor Market Flows



Note: The timing of the value functions is in orange.

Figure 4: Within period timing of the model

Figure 4 above summarizes the timing assumptions in the model. I omit time subscripts for ease of exposition. At the beginning of the period, the measures of workers with human capital h at the dominant firm, at the peripheral firm y, and in non-employment are $m_{D,-1}^h, m_{P,-1}^{h,y}$, and $m_{N,-1}^h$, respectively. The -1 subscript indicates that this distribution is inherited from the previous period. First, worker human capital evolves according to Equations (3) and (4). The resulting measures of workers with human capital h are given by

$$\hat{e}_D^h = m_{D,-1}^h(1 - g_E^h) + m_{D,-1}^{h-1} g_E^{h-1} \mathbb{1}_{\{h>h_1\}}, \tag{5}$$

$$\hat{e}_P^{h,y} = m_{P,-1}^h(1 - g_E^h) + m_{P,-1}^{h-1} g_E^{h-1} \mathbb{1}_{\{h>h_1\}}, \ \forall y \in \mathcal{P} \tag{6}$$

$$\hat{n}^h = m_{N,-1}^h(1 - g_N^h) + m_{N,-1}^{h+1} g_N^{h+1} \mathbb{1}_{\{h<h_H\}} \tag{7}$$

where $\mathbb{1}$ is the indicator function. Note $g_E^{h_H} = 0$ and $g_N^{h_1} = 0$.

Next, the birth-death process removes μ workers from the economy and μ workers are born into non-employment with $h = h_1$. The measures of employed workers at the dominant and peripheral firms after this occurs, for every $h \in \mathcal{H}$, are $e_D^h = (1-\mu)\hat{e}_D^h$ and $e_P^{h,y} = (1-\mu)\hat{e}_P^{h,y}$, $\forall y \in \mathcal{P}$. Similarly, the measure of non-employed workers with $h > h_1$ is $n^h = (1-\mu)\hat{n}^h$. A fraction μ of the non-employed with $h = h_1$ exit, but are immediately replaced. Moreover, there is an additional inflow corresponding to the exit of all other worker types in the economy. Hence the measure of non-employed workers with $h = h_1$ is

$$n^1 = (1-\mu)\hat{n}^1 + \mu \sum_h \hat{n}^h + \mu \sum_h \sum_y \hat{e}_P^{h,y} + \mu \sum_h \hat{e}_D^h. \tag{8}$$

The firms then choose wages to maximize the present discounted value of profits. I denote the aggregate distribution of workers across firms and non-employment at the time of their choice as

$$(e_P^{y_1}, \ldots, e_P^{y_M}, e_D, n) \subset \Delta^{(M+2)\times H}$$

where $e_P^y := (e_P^{1,y}, e_P^{2,y}, \ldots, e_P^{H,y})$, $\forall y \in \mathcal{P}$ is the workforce of peripheral firm y, $e_D := (e_D^1, e_D^2, \ldots, e_D^H)$ is the workforce of the dominant firm, and $n := (n^1, n^2, \ldots, n^H)$ is the distribution of non-employed workers. The firms' states are slices of this aggregate distribution, which I define in Section 2.3. Workers then decide which firm to work for or non-employment, taking expectations over the unrealized taste shocks ϵ. This mobility is characterized by conditional choice probabilities, which yield the distribution at the end of the period m_D, m_P^y, and m_N. I report this law of motion when I formalize the firm problems in Section 2.5. Finally, after matches have been consummated, production occurs both at home and on the job. Then the next period begins. I now turn to defining agents' states in the model.

2.3 State Space

Firm States. In addition to its own workforce e_D and the distribution of non-employed workers u, the dominant firm only needs to keep track of the distribution of workers across *all* peripheral firms, and not the specific split into each of the M peripheral firms. This is because peripheral firms are assumed homogeneous and atomistic.[21] I denote this as

$$s := (s^1, s^2, \ldots, s^H)$$

[21] Another reason the split into the M peripheral firms isn't a part of the dominant firm's state is that the switching cost is the same regardless of worker origin. If the switching cost was indexed by the firm, e.g. $\phi_y^{EE} \neq \phi_{y'}^{EE}$ for two different peripheral firms $y, y' \in \mathcal{P}$, then the dominant firm would need to know which peripheral firm it is poaching from.

where $s^h := \sum_{y \in \mathcal{P}} e_P^{h,y}$ is total employment of workers with human capital h at the peripheral firms. Hence, the dominant firm's state is $\chi := (e_D, u, s) \subset \Delta^{3H}$.

Worker States. The state of a worker is their human capital level $h \in \mathcal{H}$, their unobserved vector of preferences ϵ for each potential destination $d \in \{D, y_1, \ldots, y_M, N\}$, their origin $o \in \{D, P, N\}$, and χ.

2.4 Worker Value and Mobility

Worker Value. Infinitesimal relative to the economy, workers take the aggregate laws of motion as given. A worker with state (h, ϵ, o, χ) observes the wages offered to them by the dominant and peripheral firms and decides where to work. The worker's dynamic programming problem is characterized by the standard integrated value function[22]

$$V(h, o, \chi) = \int \max_d \left[u(h, o, \chi, d) + \epsilon_d + \beta(1-\mu)\mathbb{E}_{h'|h,d} V(h', d, \chi') \right] g(d\epsilon | h, o, \chi) \tag{9}$$

where $u(h, o, \chi, d)$ is the contemporaneous utility received if alternative d is chosen and $g(\cdot)$ is the probability density function of the Type 1 Extreme Value distribution.[23] For employed workers, contemporaneous utility is the sum of earnings and the amenity value of working minus the relevant switching costs:

$$\begin{aligned} u(h, o, \chi, D) &= w_D^h(\chi) + a_D^h - \phi^{NE}\mathbb{1}\{o = N\} - \phi^{EE}\mathbb{1}\{o = P\}, \\ u(h, o, \chi, P) &= w_P^h(\chi) \qquad\quad - \phi^{NE}\mathbb{1}\{o = N\} - \phi^{EE}\mathbb{1}\{o = D, P'\}, \end{aligned}$$

where $P' \neq P$ is a different peripheral firm. A non-employed worker only derives utility from their home production, $u(h, o, \chi, N) = b^h$, $\forall o$. Note that since peripheral firms are homogeneous and atomistic, in equilibrium they will post the same wages. However, the switching cost ϕ^{EE} is incurred when moving between two different peripheral firms $P \neq P'$. So for workers with $o = P$, the set of possible destinations is $d \in \{D, P, P', N\}$. Workers with $o \in \{D, P\}$ do not differentiate between P and P' and so choose from $d \in \{D, P, N\}$. Under the assumption that ϵ is Type 1 Extreme Value, the ex-ante value to a worker employed at the dominant firm has the following closed-form expression

$$V(h, D, \chi) = \sigma \Bigg(\ln \Bigg(\exp \frac{v(h, D, \chi, D)}{\sigma} + M \exp \frac{v(h, D, \chi, P)}{\sigma} + \exp \frac{v(h, D, \chi, N)}{\sigma} \Bigg) + \gamma^{EM} \Bigg)$$

[22] See Aguirregabiria and Mira (2010) for a clear survey of dynamic discrete choice models and standard results.

[23] With location normalized to 0 and scale parameter $\sigma > 0$, the pdf is $g(\epsilon_d) = \exp(-\epsilon_d/\sigma - \exp(-\epsilon_d/\sigma))/\sigma$.

where $\gamma^{EM} \approx 0.5772$ is the Euler-Mascheroni constant and

$$v(h, o, \chi, d) = u(h, o, \chi, d) + \beta(1 - \mu)\mathbb{E}_{h'|h,d} V_\sigma(h', d, \chi')$$

are the choice-specific value functions. The expressions for the value to workers at the peripheral firms and in non-employment are relegated to Appendix B.2.

Conditional Choice Probabilities. The conditional choice probability $P(d|h, o, \chi)$ is the probability that a worker with states (h, o, χ) optimally chooses destination d. To simplify notation, I write $P^h_{o\to d} = P(d|h, o, \chi)$. The set of $10 \times H$[24] choice probabilities are given by

$$P(d|h, o, \chi) = \int \mathbb{1}\{d = \underset{j\in\{D,P,N\}}{\arg\max}\, v(h, o, \chi, j) + \epsilon(j)\}g(d\epsilon|h, o, \chi), \quad o \in \{D, N\},$$

$$P(d|h, P, \chi) = \int \mathbb{1}\{d = \underset{j\in\{D,P,P',N\}}{\arg\max}\, v(h, o, \chi, j) + \epsilon(j)\}g(d\epsilon|h, o, \chi)$$

where $\mathbb{1}$ is the indicator function. The Type 1 Extreme Value assumption also gives rise to closed-form expressions for these probabilities which I report for $o = P$. A worker employed at a peripheral firm P can either remain at P, move to the dominant firm, move to another peripheral firm $P' \neq P$, or choose non-employment. Therefore, the probability of staying at the same peripheral firm is

$$P^h_{P\to P} = \frac{e^{v(h,P,\chi,P)/\sigma}}{e^{v(h,P,\chi,P)/\sigma} + e^{v(h,P,\chi,D)-\phi^{EE}/\sigma} + (M-1)e^{v(h,P,\chi,P')-\phi^{EE}/\sigma} + e^{v(h,P,\chi,N)/\sigma}}$$

I relegate the remaining probabilities, including the ones for workers starting from the dominant firm and non-employment, to Appendix B.3. Again, due to the homogeneity of the peripheral firms, we have

$$v(h, P, \chi, P') = v(h, P, \chi, P) - \phi^{EE}. \tag{10}$$

2.5 Labor Supply Curves and Firm Value

Both types of firms $i \in \{D, P\}$ choose wages $w_i = (w_i^1, \dots, w_i^H)$ every period to maximize the present discounted value of profits. The key feature of the dominant firm's problem is that

[24] For each $h \in \mathcal{H}$, there are ten conditional choice probabilities:

$$P^h_{D\to D}, P^h_{D\to P}, P^h_{D\to N}, P^h_{P\to D}, P^h_{P\to P}, P^h_{P\to P'}, P^h_{P\to N}, P^h_{N\to D}, P^h_{N\to P}, P^h_{N\to N}$$

it is *non-atomistic*. It is big enough to internalize both the effect on its labor supply curve today through the choice probabilities and the effect on its future state χ'. These two effects are intertwined and introduce a fixed point problem inside of the firm's problem: to forecast χ', the dominant firm needs to know the choice probabilities but to compute the choice probabilities, the dominant firm needs to know χ'. This subtlety arises precisely because the dominant firm is non-atomistic, a key ingredient of the model that uniquely separates it from the rest of the literature. I discuss this fixed point in more detail in Appendix C.1 and how the solution method I use allows me to avoid explicitly solving for the fixed point in Section 2.9.

The assumption that the peripheral firms behave as if they are atomistic implies that they do not internalize the effect their choice of wages has on their labor supply curve. However, importantly, this does not imply that they offer workers their marginal product of labor. Just like the dominant firm, the peripheral firms also have monopsony power due to taste shocks, switching costs, and market granularity. Even without internalizing the dynamic effects on the aggregate distribution, this still enables them to price below workers' marginal product of labor. The key difference with the dominant firm is that the wage they offer is a markdown of the *static* marginal product of labor, which I discuss more in Section 2.6. Therefore, the dominant firm is a dynamic monopsonist and the peripheral firms are static monopsonists.

Labor Supply Curves. I denote by $m_i := (m_i^1, \ldots, m_i^H)$ the labor supply curve faced by firm $i \in \{D, P\}$ where m_i^h is the mass of workers with human capital h that decide to come to firm i. The labor supply curve firms face depends on their choice of wages. An important measure of i's market power is thus its labor supply elasticity,

$$\varepsilon_i^h(\chi, w_i) := \frac{\partial m_i^h(\chi, w_i)}{\partial w_i^h} \frac{w_i^h}{m_i^h(\chi, w_i)}, \tag{11}$$

which captures how much employment of workers with human capital h at firm i changes following a change in wages offered.

For the dominant firm, the labor supply curve for each worker type h is

$$m_D^h(\chi, w_D) = \underbrace{e_D^h P_{D \to D}^h(\chi, w_D)}_{\text{retained}} + \underbrace{s^h P_{P \to D}^h(\chi, w_D)}_{\text{poached from } P} + \underbrace{n^h P_{N \to D}^h(\chi, w_D)}_{\text{hired from } N}, \tag{12}$$

where the first term corresponds to retained workers, the second term to workers poached from rival peripheral firms, and the last term to workers hired from non-employment. The labor supply curves of the peripheral firms are similar,

$$\begin{aligned} m_P^h(\chi, w_P) = &\underbrace{e_D^h P_{D\to P}^h(\chi, w_P)}_{\text{poached from } D} + \underbrace{n^h P_{N\to P}^h(\chi, w_P)}_{\text{hired from } N} \\ &+ \underbrace{e_P^{h,y} P_{P\to P}^h(\chi, e_P, w_P)}_{\text{retained}} + \underbrace{s_P^{h,y} P_{P'\to P}^h(\chi, w_P)}_{\text{poached from } P' \neq P}, \end{aligned} \tag{13}$$

with the main difference being an additional term that captures hiring from other peripheral firms. Notice only the retained workers do not pay the switching cost ϕ^{EE}. Hence, an inefficiency is introduced by imposing firms offer the same wages to workers regardless of origin. I discuss this more when I characterize the centralized equilibrium in Section 3.

Another implication of the dominant firm being non-atomistic is it not only internalizes the effect on its own labor supply curve but also on the labor supply curves of its rivals and the distribution of workers who choose non-employment. I define $m_N := (m_N^1, \ldots, m_N^H)$ where the mass of workers with human capital h who choose non-employment is

$$m_N^h = \underbrace{e_D^h P_{D\to N}^h}_{\text{inflow from } D} + \underbrace{s^h P_{P\to N}^h}_{\text{inflow from } P} + \underbrace{n^h P_{N\to N}^h}_{\text{remaining at } N} . \tag{14}$$

After matches are formed, human capital changes and these labor supply curves become χ' according to the laws of motion presented in Section 2.2.

Firm Problem. We are now ready to formalize the dominant firm's problem. The dominant firm's value, $J_D(\chi)$, solves the following Belllman equation

$$J_D(\chi) = \max_{w_D \in \mathbb{R}_+^H} F_D(m_D(\chi, w_D)) - \sum_h m_D^h(\chi, w_D) \times w_D^h + \beta \mathbb{E} J_D(\chi') \tag{15}$$

subject to the labor supply curves given by Equations (12) to (14) and the law of motion for $\chi'(\chi, w_D)$ given by Equations (5) to (8). The first two terms are contemporaneous revenue: output less the wage bill. The last term is the continuation value where the expectation is taken over the human capital accumulation of all the workers in the market. I relegate the first-order conditions, which I do not explicitly use to solve the model, to Appendix B.4 and a more formal discussion of the mechanics of the dominant firm being non-atomistic to Appendix B.5.

The main tradeoff in the firm's problem is between present and future profits. On the one hand, there is an incentive for the dominant firm to exploit its monopsony power and choose low wages, thus increasing today's profits. On the other hand, lowering wages affects the sorting of workers and their subsequent human capital accumulation. Some workers will opt to go to the peripheral firms, while others will choose non-employment. Both will be more expensive to re-hire in the future because of the switching costs and in addition, the workers going to non-employment will see their human capital depreciate. Hence, by lowering wages today, the dominant firm hinders future profits.

The extent to which they exploit this market power depends on how much market power they have to begin with, as well as the learning environment in the market. I study these tradeoffs more formally and quantify their implications in Section 5.

2.6 Dynamic Markdowns

The wage equation for the dominant firm, derived in Appendix B.4, is

$$w_D^h = \underbrace{\frac{\varepsilon_D^h(\chi, w_D)}{\varepsilon_D^h(\chi, w_D) + 1}}_{\text{markdown}} \left[\underbrace{\frac{\partial F_D(\chi, w_D)}{\partial m_D^h(\chi, w_D)}}_{\text{static term}} + \underbrace{\beta \mathbb{E} \frac{\partial J_D(\chi')}{\partial m_D^h(\chi, w_D)}}_{\text{dynamic term}} \right]. \tag{16}$$

The gap between the wage and the marginal product of labor is the wage markdown, which I denote η_D^h for the dominant firm, and is equal to the Lerner index (Lerner 1934) of the firm's labor supply elasticity. The marginal product of labor is the sum of a static component and a dynamic component. The static term captures how much output at the dominant firm today changes as it hires an additional worker with human capital h. The dynamic term captures the impact on the firm's value tomorrow of hiring a worker today: this additional worker will be both more skilled and less costly to hire.

Equation (16) nests the standard static wage markdown equation. The most straightforward way to recover a static markdown is if there are no dynamics and the firm maximizes period profits (i.e. $\beta = 0$). Moreover, if the firm is atomistic and does not internalize the effect its choice of wages has on its future state, then the dynamic term also disappears. It is not only the fact that the firm is foward-looking that matters, but also that it is big enough to internalize its effect on the aggregate state. The wage equation for the peripheral firms, which I assume behave as if they were atomistic, is therefore

$$w_P^h = \underbrace{\frac{\varepsilon_P^h(\chi, w_P)}{\varepsilon_P^h(\chi, w_P) + 1}}_{\text{markdown}} \frac{\partial F_P(\chi, w_P)}{\partial m_P^h(\chi, w_P)}, \tag{17}$$

where I show in Corollary 1 that the static markdown,

$$\eta_P^h := \frac{\varepsilon_P^h(\chi, w_P)}{\varepsilon_P^h(\chi, w_P) + 1},$$

is equal to the gap between wages in the centralized and decentralized equilibria. I discuss in Section 4.2 how I estimate $\varepsilon_P^h(\chi, w_P)$ internally which I then use to solve for the equilibrium of the model. Notice that the model admits a distribution of markdowns across firms, i.e. between the dominant and peripheral firms, as well as within firms, i.e. for different human capital levels.

There are many ways in which the static markdown equation can be extended to allow for dynamics and in fact, this paper is not the first to do so. Wong (2023) extends the seminal work of Manning (2006) to allow for imperfect competition in the goods market as well as a richer production technology and also obtains a dynamic markdown equation that features the marginal profit associated with the hiring of an additional worker.

In static models, the markdown is a sufficient statistic for the welfare losses due to monopsony power because the efficient outcome is to offer workers their marginal product. However, as pointed out by Manning (2003), it is not clear if the markdown remains the relevant measure in more general settings, which is why why in Section 3 I study the centralized equilibrium of the model. In Theorem 1, I show that the *static* marginal product of labor decentralizes the efficient allocation. Therefore, the *dynamic* markdown (and the Lerner index) in Equation (16) is not a sufficient statistic for the welfare losses.

2.7 Equilibrium Concept

I study stationary competitive equilibria of the model. The stationarity restriction requires that firm policies are not indexed by time t. Stationarity does not change the fact that the strategic environment firms compete in is inherently time-varying through the dynamics. It merely restricts next period's aggregate distribution implied by firms' strategies to coincide with the current distribution, i.e. $\chi' = \chi$. In fact, as I discuss in Appendix C, I solve for the recursive equilibrium and then impose stationarity.

Having justified the main restriction on the class of equilibria studied, we can now formally define an equilibrium in this economy:

Definition 1 (Equilibrium). *A stationary competitive equilibrium is a worker value $\{V(h, o, \chi)\}$, firm values $\{J_D(\chi), J_P(\chi)\}$, firm policies $\{w_D(\chi), w_P(\chi)\}$ and a law of motion for χ such that*

1. *The worker value satisfies Equation (9) given firm policies and the law of motion for* χ*.*

2. $J_D(\chi)$ *solves Equation (15) given worker values and the law of motion for* χ*.*

3. $w_P(\chi)$ *solves Equation (17) given worker values and the law of motion for* χ*.*

4. *The law of motion for* χ *given by Equations (5) to (8) and (12) to (14) is consistent with the worker value function and firm policies.*

5. *The aggregate distribution of workers across firms and non-employment is stationary:*

$$\chi' = \chi.$$

Existence and Uniqueness. Because I assume the peripheral firms are atomistic, the strategic interaction between the dominant firm and the peripheral firms is trivial. The peripheral firms always offer workers a constant *static* markdown so no best response iteration is necessary. Nonetheless, some results from the dynamic oligopoly literature still apply here. In dynamic pricing games with *single-product* firms[25] where the strategy sets are convex and compact and the profit function is quasi-concave, there exists a pure strategy Nash equilibrium which is obtained by applying the Brouwer or Kakutani fixed point theorems (Caplin and Nalebuff 1991). This approach does not generally extend to settings with *multi-product* firms[26] because it is no longer guaranteed that firm profits are quasi-concave (Hanson and Martin 1996). This issue is present here as well: Lemma 1 in Appendix B.6 shows that the dominant firm's value function, Equation (15), is neither concave (sufficiency of first-order conditions) nor quasi-concave (existence). Finally, the removal of strategic interactions also helps bypass issues of multiplicity common in this class of models. In practice, when I solve the model I do not run into multiple equilibria.

2.8 Discussion

I now discuss several key modeling choices and potential extensions before turning to the planner's problem.

[25] This is equivalent to the nested case $H = 1$ where all workers are homogenous, effectively shutting off human capital accumulation in the model.

[26] The model with $H > 1$ is analogous to a dynamic oligopoly model with multi-product firms. There are two key differences. First, here the strategic interactions are trivial since the peripheral firms are atomistic. Second, here the *supply* of workers is dynamic, whereas in multi-product firm models, the *demand* for products is static.

Human Capital. The human capital accumulation process I study is stylized and can be extended along several dimensions. First, human capital only accumulates through the extensive margin, i.e. there is no hours decision on the worker side. I do not allow for an intensive margin because the variance of hours in the data is small. Workers in France are subject to the 35-hours workweek policy enacted in 1998.[27] Moreover, since the data is constructed from the DADS forms which firms fill out annually, it is reported hours and not actual hours that are observable. Put together, these two features imply a large mass point of weekly hours worked at 35 with little variation.

Second, I focus on general human capital. Firm-specific human capital would give firms additional market power: with non-transferrable human capital, the firms' workforces will be more persistent over time. However, extending the model to allow for this would entail an increase in the dimensionality of the worker and firm problems.[28] Third, I do not allow for firm-specific learning rates, i.e. the rate at which workers learn while employed is the same across firms. It is straightforward to add this in the model: g_E^h would be split into g_D^h and g_P^h. This would add an extra aspect of differentiation of firms from the worker's perspective, giving them more monopsony power.

Segmented Markets. This is a model of segmented markets. There are many ways to extend the model to allow for mobility across markets, but perhaps the most natural would be to use a sequential decision structure such as the nested logit. Workers would first choose which labor market to participate in and then, conditional on the choice of market, a firm or non-employment. However, without imposing a behavioral assumption,[29] this would render the model intractable since firms would be competing not only with the other firms in their market but also with all the other firms across all the other markets. A more feasible setup would be to allow firms to be non-atomistic within markets, but atomistic across markets as in Berger, Herkenhoff, and Mongey (2022). Since I define a local labor market to be the combination of a region and a 4-digit occupation code, extending the model to allow for mobility across markets would entail modeling either commuting zone or occupation switching. Both would alter the tradeoffs the firms face described in Section 2.5.

[27] See Estevão et al. (2008) for a clear history of hours reductions in France since the 1980s.

[28] Suppose workers are also heterogeneous in firm-specific human capital that takes values in a discrete grid of size F. Then the aggregate state would now be the *joint* distribution of workers across both types of human capital, firms, and non-employment. Assuming the same market structure, this would have cardinality $3 \times H \times F$.

[29] For example, one could impose that firms do not internalize the effect their choices have on the sorting of workers in other markets.

Market Power. The extent of monopsony power firms have is governed by several parameters. First, firms derive market power from the granularity of the market, captured by the number of firms $M + 1$. All else equal, increasing M makes the market more competitive. Second, preference heterogeneity induced by the logit taste shocks creates classic differentiation between the firms and non-employment. These reflect non-wage amenities such as working conditions or commuting costs that are specific to the firm. Since I normalize the location of the shocks to 0, the scale σ determines the strength of this effect. Third, the relative amenity value of being employed at the dominant firm $a_D := (a_D^1, a_D^2, \dots, a_D^H)$ creates classic differentiation between both firm types. These reflect differences in preferences for the dominant firm and *any* of the peripheral firms. Finally, the moving costs ϕ^{EE} and ϕ^{NE} also differentiate firms from the point of view of the worker and give the incumbent firm monopsony power.

A popular alternative approach to modeling monopsony power is with search frictions. The most influential example is Burdett and Mortensen (1998), on which recent several recent papers build (Engbom and Moser 2022, Gouin-Bonenfant 2022, Berger, Herkenhoff, Kostøl, and Mongey 2023, and Gottfries and Jarosch 2023). In any search and matching model of the labor market, firms reap the benefits of an upward-sloping labor supply curve because of costly reallocation from the search frictions. The switching costs are isomorphic to search frictions but are not endogenous.

Market Structure. The market configuration in this paper, one dominant firm and a finite number of peripheral firms (fringe), is common in dynamic oligopoly models (e.g. Benkard et al. 2015) and is chosen to mimic the features of the data described in Section 1.3. The main restriction in terms of market structure is that the peripheral firms are homogeneous which implies they have equal employment in equilibrium. Nonetheless, the model nests a diverse range of market structures. For example, the model reduces to perfect competition if $p_D = 1$, $\phi^{EE} = \phi^{NE} = 0, M \to \infty$ and a_D^h for all h. Fixing p_D, increasing the number of peripheral firms M will yield less and less concentrated markets.

2.9 Solution Method

The aggregate state, shared by both workers and firms in the model, is the distribution of workers across firm types and human capital levels and has cardinality $3 \times H$. Even for relatively small H, the curse of dimensionality (Bellman 1957) kicks in quickly. Suppose $H = 3$ and we solve the model on a very coarse grid with the 11 points $\{0, 0.1, 0.2, \dots, 1\}$. Since $\chi \subset \Delta^{3H}$, we would not have to solve the model for each of the $11^9 \approx 2.3$ billion combinations, but "only" for combinations that sum to 1. A simple application of the stars and bars theorem in combinatorics shows that there are still tens of thousands of combinations to consider, making this intractable. So even solving the model on a very coarse grid for a small number of human capital levels is not possible. [30]

Hence, I solve for an equilibrium by approximating the key model objects: the firm policies and values, the worker value, and the laws of motion. Polynomials (e.g. Chebyshev or Hermite), the conventional class of approximating functions, are not well-suited to this setting. They cannot perform the dimension reduction necessary to handle such a large state space, e.g. updating the coefficients on the series approximating the firm policies would require directly solving for the optimal policies a prohibitively large number of times. They are also known to have explosive behavior when extrapolating outside the bounds of the fitted area and have difficulty fitting local features.

To bypass these issues, I use deep neural networks. I relegate a summary of neural networks to Appendix C.2, but discuss several of their properties here. First, the universal approximation theorem (Hornik et al. 1989) states that neural networks with at least one hidden layer can get arbitrarily close to any continuous function. Remarkably, neural networks can approximate highly nonlinear applications such as binding constraints, kinks and ridges, non-differentiabilities, discontinuities, and discrete choices. Second, neural networks circumvent the curse of dimensionality. Barron (1993) shows that the number of neurons required to approximate a function scales *linearly* with the number of dimensions of the input. In contrast, this grows exponentially for series approximations. Third, recent advances in computing have led to the development of state-of-the-art machine learning libraries (e.g. Google's TensorFlow), which allow neural networks to be estimated very quickly.

More precisely, I use deep reinforcement learning, which refers to the use of reinforcement learning algorithms with deep neural networks. I relegate an overview of deep reinforcement learning and a detailed description of the solution method to Appendix C.4, but provide here a high-level discussion of its main benefit: solving for the firm's optimal choice of wages without resorting to any direct optimization. In prediction problems, the parameters of a neural network are commonly updated to minimize the loss from inaccurate predictions.[31] Using such a loss function for the firm problem entails finding the optimal policy for each of the training points at each iteration. If done with an optimizer,[32] this requires an overwhelming amount of evaluations of the firm profits. To avoid this, I modify the deep deterministic

[30] Interpolation in high dimension can also be problematic, both in terms of accuracy and in terms of speed.

[31] The most common loss function is the mean squared error (MSE) $\frac{1}{N_t}\sum_i(\hat{y}_i - y_i)^2$ where $\hat{y}_i$ is the predicted value, y_i is the truth, and N_t is the number of points in the training data.

[32] As discussed in Section 2.7 and shown in Appendix B.6, the dominant firm's value is neither concave nor quasi-concave. Therefore sufficiency of the first-order conditions is not guaranteed. For this reason, I do not use the firm's first-order conditions to characterize its optimal choice of wages.

policy gradient (DDPG) method proposed by Silver et al. (2014) and Lillicrap et al. (2015). This is an actor-critic algorithm, a popular method in deep reinforcement learning in which both the policy (actor) and the value (critic) are approximated by neural networks. The neural network weights for the value are updated by minimizing the Bellman residuals. For the policy, the neural network weights are updated by minimizing the *negative* value function (equivalent to maximizing the value).[33]

3 The Centralized Equilibrium

Does allowing for human capital accumulation exacerbate or mitigate the inefficiencies from firms having monopsony power? To better understand the welfare costs of monopsony in the presence of dynamic human capital accumulation, in this section I describe the centralized equilibrium of the model. I first define the planner's problem and discuss model inefficiencies then characterize the wages that decentralize the efficient allocation. In Section 5, I use the estimated model to quantify the welfare losses.

3.1 The Planner's Problem

Consider a social planner that chooses the allocation of workers across firms to maximize market-wide welfare. Consistent with the decentralized equilibrium, I assume the planner's problem occurs at the same time as firms choose wages, i.e. after human capital changes and before workers receive their taste shocks (see Figure 4 for model timing). Because of the switching costs, the aggregate distribution of human capital is not sufficient and the planner needs to know the distribution of workers across the dominant firm, the peripheral firms, and non-employment. His state is thus also $\chi = (e_D, u, s) \subset \Delta^{3\times H}$. The planner simultaneously chooses the allocation of all worker types to a destination d. In the model, workers go where they receive the most utility. Here, the planner chooses a value to offer each worker type (h, o, ϵ) so as to affect their mobility choice. Notice that since the utility value workers receive from their idiosyncratic taste shock ϵ affects their choice of destination, the planner's choice of value is indexed by ϵ. As in Lagakos et al. (2023) and Waugh (2023), this is equivalent to the planner directly choosing the $10 \times H$ probabilities

$$\{P^h_{D\to d}\}_{d\in\{D,P,N\}}, \{P^h_{P\to d}\}_{d\in\{D,P,P',N\}}, \{P^h_{N\to d}\}_{d\in\{D,P,N\}}, \ \forall h \in \mathcal{H},$$

[33] The lifetime reward approach of Maliar et al. (2021) also updates the neural network for the policy using the negative value as the loss, but instead of approximating the value, they use simulation. Given predicted policies, they simulate forward to obtain a finite-horizon (truncated) approximation of the value function.

which I collapse into $\mathbb{P} \subset [0,1]^{10\times H}$. Note these are probabilities so there are the same three associated constraints that ensure they sum to 1 as in the decentralized equilibrium.

The planner's choice of $\mathbb{P}$ determines worker mobility within the period. The number of workers with human capital h the planner sends to the dominant firm is

$$m^h_{D,S}(\mathbb{P}) := e^h_D P^h_{D\to D} + s^h P^h_{P\to D} + n^h P^h_{N\to D}. \tag{18}$$

Similarly, the number of workers with human capital h who get placed in non-employment is

$$m^h_{N,S}(\mathbb{P}) := e^h_D P^h_{D\to N} + s^h P^h_{P\to N} + n^h P^h_{N\to N}. \tag{19}$$

Next, the inflow of workers at any of the M peripheral firms includes workers coming from the dominant firm and non-employment, workers remaining at the peripheral firm, and workers coming from the other $M-1$ peripheral firms is

$$m^h_{P,S}(\mathbb{P}) := e^h_D P^h_{D\to P} + \frac{s^h}{M} P^h_{P\to P} + \frac{M-1}{M} s^h P^h_{P'\to P} + n^h P^h_{N\to P}. \tag{20}$$

Since the planner allocates all workers somewhere, we have the adding-up constraint $\sum_h m^h_{D,S}(\mathbb{P}) + m^h_{N,S}(\mathbb{P}) + M m^h_{P,S}(\mathbb{P}) = 1$. Finally, because of the switching costs, it is also convenient to define the number of workers with human capital h who make a transition from one firm to another,

$$m^h_{EE,S}(\mathbb{P}) := e^h_D P^h_{D\to P} + s^h P^h_{P\to D} + s^h (M-1) P^h_{P\to P'}, \tag{21}$$

as well as the number of workers who transition out of non-employment,

$$m^h_{NE,S}(\mathbb{P}) := n^h P^h_{N\to D} + n^h P^h_{N\to P}. \tag{22}$$

To simplify notation, as before, I collapse these into the H-dimensional vectors $m_{D,S}$, $m_{N,S}$, $m_{P,S}$, $m_{EE,S}$, and $m_{NE,S}$.

The utility workers derive from the taste shocks is determined by the planner's choice. Hence, a key component of the planner's objective is the *ex-ante* (expected) value of the taste shock for all workers. The Type 1 Extreme Value assumption yields a closed-form expression for this object, commonly referred to as the conditional choice expected utility (see Aguirregabiria 2021 section 11.2 for a derivation). Given a transition probability $P^h_{o\to d}$

chosen by the planner, the expected value of the worker's preference shock is given by

$$\mathbb{E}[\epsilon | P^h_{o\to d}] = \sigma\gamma^{EM} - \sigma \ln P^h_{o\to d} \tag{23}$$

where, again, γ^{EM} is the Euler-Mascheroni constant. Aggregating over all workers, the total ex-ante value of these shocks given the planner's choice of $\mathbb{P}$ is

$$\begin{aligned}
\mathcal{E}(\mathbb{P}) := &\sum_h e^h_D \Big(P^h_{D\to D}\mathbb{E}[\epsilon|P^h_{D\to D}] + MP^h_{D\to P}\mathbb{E}[\epsilon|P^h_{D\to P}] + P^h_{D\to N}\mathbb{E}[\epsilon|P^h_{D\to N}] \Big) \\
&+ \sum_h s^h \Big(P^h_{P\to D}\mathbb{E}[\epsilon|P^h_{P\to D}] + P^h_{P\to P}\mathbb{E}[\epsilon|P^h_{P\to P}] + (M-1)P^h_{P\to P'}\mathbb{E}[\epsilon|P^h_{P\to P'}] + P^h_{P\to N}\mathbb{E}[\epsilon|P^h_{P\to N}] \Big) \\
&+ \sum_h n^h \Big(P^h_{N\to D}\mathbb{E}[\epsilon|P^h_{N\to D}] + MP^h_{N\to P}\mathbb{E}[\epsilon|P^h_{N\to P}] + P^h_{N\to N}\mathbb{E}[\epsilon|P^h_{N\to N}] \Big).
\end{aligned}$$

The first term is the total value of the shocks for workers that begin the period employed at the dominant firm. There are e^h_D of these workers with human capital h. Of those, a fraction $P^h_{D\to D}$ will remain at the dominant firm, a fraction $MP^h_{D\to P}$ will move to a peripheral firm, and a fraction $P^h_{D\to N}$ will move to non-employment. Within each of these groups, workers will receive different vectors of taste shocks, but their expected taste shock will be the same and given by Equation (23). The second term and last term are analogous expressions for workers starting the period employed at peripheral firms and non-employed, respectively.

We are now ready to formally define the planner's problem. The planner's value, $S(\chi)$, solves the following Bellman equation:

$$\begin{aligned}
S(\chi) = \max_{\mathbb{P}\in[0,1]^{10\times H}} & F(m_{D,S}(\mathbb{P})) + MF(m_{P,S}(\mathbb{P})) + \sum_h m^h_{N,S}(\mathbb{P})b(h) + \mathcal{E}(\mathbb{P}) \\
&+ \sum_h m^h_{D,S}(\mathbb{P})a^h_D - \phi^{EE}\sum_h m^h_{EE,S}(\mathbb{P}) - \phi^{NE}\sum_h m^h_{NE,S}(\mathbb{P}) + \beta\mathbb{E}S(\chi')
\end{aligned} \tag{24}$$

subject to Equations (18) to (22), the law of motion for χ' given by Equations (5) to (8), and the three adding up constraints for $\mathbb{P}$. The first three terms of the planner's objective are market-wide output, the fourth is the total value of the taste shocks, the sixth is the total amenity values, and the last two are the moving costs paid by movers. Like the dominant firm's problem, the planner's problem is dynamic.

3.2 Inefficiencies

There are two inefficiencies. First, there is an allocative efficiency stemming from the restriction that firms cannot price discriminate according to a worker's origin. The wage offered to a worker with human capital h is the same whether the worker is retained, poached from a rival firm, or hired from non-employment even though these workers are subject to different switching costs. Therefore, firms in the model do not fully internalize the benefits to the workers of employment at other firms. Second, as in Mongey and Waugh (2023), an additional inefficiency arises because markets are incomplete: workers cannot insure against their idiosyncratic taste shocks. I focus solely on allocative efficiency and will have nothing to say about the second inefficiency. In this regard, the planner is constrained: they can control the allocations of workers across the various firms and non-employment, but can not provide the missing market of insurance

3.3 Decentralization

I now characterize the dominant wages that decentralize the efficient allocation of workers to firms and non-employment, which I denote $w^h_{D,S}$ and $w^h_{P,S}$.

Theorem 1 (Decentralization)**.** *The static marginal product of labor decentralizes the efficient allocation, both for the dominant firm,*

$$w^h_{D,S} = p_D \times h, \tag{25}$$

and for the peripheral firms,

$$w^h_{P,S} = h. \tag{26}$$

Proof. First, since production, given by Equation (2), is constant returns to scale, the planner's problem is separable. The elasticity of substitution between different worker types is 1, therefore there are no spillovers in production between different workers. Instead of jointly choosing the allocation of workers, the planner can choose where to allocate each worker type (h, ϵ, χ) where $\epsilon = \{\epsilon_D, \epsilon_{y_1}, \epsilon_{y_1}, \dots, \epsilon_{y_M}, \epsilon_N\}$. Equation (24) can thus be expressed as the distribution weighted sum of worker-level values,

$$S(\chi) = \sum_{h\in\mathcal{H}} \sum_{o\in\{D,P,N\}} m(h, o, \chi) S(h, o, \chi), \tag{27}$$

where the weight $m(h, o, \chi)$ is the number of workers of type (h, o) with aggregate state χ.

As with the worker value Equation (9), $S(h, o, \chi)$ is the integrated value to the planner:

$$S(h, o, \chi) = \int \max_d \left[u_S(h, o, \chi, d) + \epsilon_d + \beta(1-\mu)\mathbb{E}_{h'|h,d} S(h', d, \chi') \right] g(d\epsilon | h, o, \chi). \tag{28}$$

$u_S(h, o, \chi, d)$ is the contemporaneous welfare (excluding the realization of the taste shock ϵ_d) resulting from a worker of human capital h with origin o going to a destination d:

$$\begin{aligned}
u_S(h, o, \chi, D) &= \frac{\partial F_D}{\partial m_D^h} + a_D^h - \phi^{NE}\mathbb{1}\{o = N\} - \phi^{EE}\mathbb{1}\{o = P\}, \\
u_S(h, o, \chi, P) &= \frac{\partial F_P}{\partial m_P^h} \qquad\quad - \phi^{NE}\mathbb{1}\{o = N\} - \phi^{EE}\mathbb{1}\{o = D, P'\}, \\
u_S(h, o, \chi, N) &= b^h.
\end{aligned}$$

$\frac{\partial F_i}{\partial m_i^h}$ is the *static* marginal product of labor, e.g. $\partial F_D / \partial m_D^h$ is how much the output of the dominant firm changes when it has one more worker with human capital h. With linear production, the marginal products are $\frac{\partial F_D}{\partial m_D^h} = p_D \times h$ and $\frac{\partial F_P}{\partial m_P^h} = h$ since p_P is normalized to 1. Comparing Equation (28) with Equation (9), it is clear that setting the wages equal to the static marginal product decentralizes the efficient allocation:

$$\begin{aligned}
w_{D,S}^h &= \frac{\partial F_D}{\partial m_D^h} \implies w_{D,S}^h = p_D \times h \\
w_{P,S}^h &= \frac{\partial F_P}{\partial m_P^h} \implies w_{P,S}^h = h
\end{aligned}$$

This concludes the proof. □

This result is reminiscent of the seminal work of Becker (1962). With constant returns to scale production, the destination choice of any one worker does not impact either pay or productivity of any other worker in the economy. In other words, the worker's choice does not have any spillovers to other workers. So if workers receive the full value of the output they produce while employed, then they internalize the full set of costs and benefits of their decisions. Hence offering them their *static* marginal product of labor results in the efficient allocation. Perhaps surprisingly, there is no dynamic component. The reason for this is two-fold. One contributing factor is that the worker is forward-looking and already internalizes the present discounted value of wages they can make in the future. Additionally, the worker is atomistic so does not internalize how their choice can affect the aggregate distribution.

Next, I define ν_i^h for $i \in \{D, P\}$ and $h \in \mathcal{H}$ to be the ratio between the wages that decentralize the planner's problem and the wages offered in the decentralized equilibrium.

Corollary 1. *ν_P^h is equal to the wage markdown η_P^h:*

$$\nu_P^h := w_P^h / w_{P,S}^h = w_P^h / h.$$

Corollary 1 follows immediately from Theorem 1 and is the standard result found in static models of monopsony: the markdown $\eta_P^h = \varepsilon_P^h(\chi, w_P)/(\varepsilon_P^h(\chi, w_P) + 1)$ is a sufficient statistic to quantify the welfare losses due to monopsony. In Section 4.2, I describe how I estimate $\varepsilon_P^h(\chi, w_P)$ internally which I then use to compute η_P^h and solve the model.

Corollary 2. *ν_D^h is not equal to the wage markdown η_D^h:*

$$\nu_D^h := w_D^h / w_{D,S}^h = w_D^h / (p_D \times h).$$

Corollary 2 also follows immediately from Theorem 1. There are several takeaways. First, ν_D^h is decreasing in the productivity gap. Note that $p_D = 1$ does not imply that the market is perfectly competitive. The wages the dominant firm offers w_D^h are below the worker's marginal product even if $p_D = 1$ because it still has monopsony power from the taste shocks, switching costs, amenities, and granularity of the market. Moreover, although the wages that decentralize the efficient allocation are static, ν_D^h is dynamic: w_D^h is set according to the wage equation given by Equation (16) which contains a dynamic term arising due to the human capital accumulation of workers. Lastly, ν_D^h is *not* equal to the markdown $\eta_D^h = \varepsilon_D^h(\chi, w_D)/(\varepsilon_D^h(\chi, w_D) + 1)$. Substituting in Equation (16) yields and expression for ν_D^h in terms of the markdown and the Lerner index:

$$\nu_D^h = \underbrace{\frac{\varepsilon_D^h(\chi, w_D)}{\varepsilon_D^h(\chi, w_D) + 1}}_{:= \eta_D^h} \left[1 + \frac{\beta \mathbb{E} \frac{\partial J_D(\chi')}{\partial m_D^h(\chi, w_D)}}{p_D \times h}\right]. \tag{29}$$

If the dynamic component of the marginal product is large, either positive or negative, then the markdown is very far from capturing the welfare losses. As mentioned in Section 2.6, the usual static case where $\nu_D^h = \eta_D^h$ can be recovered if the dynamic term is zero, which can occur if there is no discounting, $\beta = 0$, or if the dominant firm is atomistic and does not internalize its effect on the aggregate distribution χ'.

This gap between ν and η has important implications for the measurement of monopsony power. In static models where $\eta = \nu$, there are two approaches used to estimate the markdown. Researchers can either measure the elasticity of labor supply or measure the marginal product of labor, i.e. either of the two terms in the Lerner formula above. Once we allow for dynamics, the markdown η is no longer equal to the distance ν to the efficient outcome because of the additional dynamic component. So even if we could accurately measure the markdown η, it does not tell us anything about the welfare losses. Moreover, markdown estimates obtained using first-order conditions of a static firm problem are misspecified and risk biasing estimates by ignoring the dynamics.

4 Estimation

I estimate the model parameters using the DADS-Panel because the longitudinal dimension is needed to measure human capital. I first discuss identification, then present parameter estimates, and finally assess model fit.

4.1 Externally Set

I externally calibrate $\{\beta, \mu, M, H\}$. The model is estimated at a yearly frequency so I set the discount factor β to 0.95.[34] The birth-death rate μ is set to $1/40 = 0.025$ so that the average worker is in the labor force for 40 years. Next, the number of firms M in a market is observable. Finally, I choose $H = 3$ levels of human capital. Although this helps with tractability, the solution method can handle much larger values of H.[35] The reason I do not allow for more human capital levels is because of the small sample size. Estimation requires clustering worker-year observations into H different human capital levels and to be model consistent each market must have at least one worker of each type in the dominant firm, the peripheral firms, and non-employment. Using the DADS-Panel means losing approximately $11/12th$ of the observations hence I am left with some markets that do not have enough workers per year to satisfy these clustering requirements. Increasing H would require only considering very large markets.

4.2 Internally Estimated

There are 14 remaining parameters to estimate:

$$\theta := \{b^1, b^2, b^3, \phi^{EE}, \phi^{NE}, \sigma, g_E^1, g_E^2, g_N^3, g_N^2, a_D^1, a_D^2, a_D^3, p_D\}.$$

Note that the human capital grid $\mathcal{H}$ does not need to be estimated since it can be inferred from $w_P^h = \eta_P^h \times h$ conditional on knowing η_P^h. I distinguish between the supply-side param-

[34] This is roughly equivalent to an annual interest rate of $5\% : 1/(1 + 0.05) \approx 0.952$.

[35] Recent papers in economics using neural networks can handle thousands of states. For example, Maliar et al. (2021) solve a variant of a Krusell and Smith (1998) problem with 1,000 agents which implies 2,001 state variables.

eters,

$$\theta_S := \{b^1, b^2, b^3, \phi^{EE}, \phi^{NE}, \sigma, g_E^1, g_E^2, g_N^3, g_N^2, a_D^1, a_D^2, a_D^3\},$$

and the sole demand side parameter, p_D. Given firm policies, θ_S alone determines worker mobility. Given θ_S, firm behavior only depends on p_D. This structure allows me to use a two-stage procedure commonly used in the dynamic discrete choice literature to estimate the remaining model parameters.

Supply. I first estimate θ_S in partial equilibrium, independent of p_D. Suppose a discretization of the data is known so that we observe the contracts $w^{\text{data}} = (w_D^{\text{data}}, w_P^{\text{data}})$, the $10 \times H = 30$ conditional choice probabilities stacked into P^{data}, and the $3 \times H = 9$ dimensional distribution of workers across firm types and human capital levels χ^{data}. Then θ_S can be estimated using indirect inference:

$$\hat{\theta}_S = \arg\min_{\theta_S} \sum_{i=1}^{30} \left(\frac{P_i^{\text{data}} - P_i(\theta_S; w^{\text{data}}, \chi^{\text{data}})}{P_i^{\text{data}}} \right)^2 + \sum_{i=1}^{9} \left(\frac{\chi_i^{\text{data}} - \chi_i'(\theta_S; w^{\text{data}}, \chi^{\text{data}})}{\chi_i^{\text{data}}} \right)^2$$

The first term is the distance between P^{data} and the model implied conditional choice probabilities $P(\theta_S; w^{\text{data}}, \chi^{\text{data}})$. This pins down the parameters governing mobility across the different firms and non-employment. The second target is the law of motion for the aggregate state χ' which contains information on the human capital transitions of workers from one period to the next. In addition, this imposes the steady state assumption: $\hat{\theta}_S$ is chosen to minimize the distance between the distribution today χ^{data} and $\chi_i'(\theta_S; w^{\text{data}}, \chi^{\text{data}})$, the distribution next period implied by the model given wages w^{data} and χ^{data}.[36]

To find the global minimum, I first sample 50 million Sobol points from the parameter space and then use the parameter vector yielding the lowest objective value as an initial guess for a local optimizer.[37] In Appendix D.1, I plot slices of the objective function to illustrate that the parameters are locally identified.

Peripheral Markdowns. Next, given $\hat{\theta}_S$, I estimate the markdowns offered by the peripheral firms. Recall that the peripheral firms are *static* monopsonists and price according to Equation (17), i.e. $w_P^h = \eta_P^h \times h$ where $\eta_P^h = \nu_P^h$ by Corollary 1. For all human capital levels $h \in \mathcal{H}$, I estimate the peripheral firms' labor supply elasticity ε_P^h as the change in employment at a peripheral firm following a 1% decrease in wages offered. I assume this change is both unexpected and does not persist for more than one period. The markdown is

[36] In practice, I use the distribution five periods ahead χ''''' to further impose stationarity.

[37] I use a combination of the Nelder-Mead (Nelder and Mead 1965) and Subplex (Rowan 1990) algorithms to polish off the Sobol minimizer.

then simply $\eta_P^h = \varepsilon_P^h/(\varepsilon_P^h + 1)$. Since wages at the peripheral firms are observable, this pins down the human capital grid which I use to solve for the general equilibrium of the model in the second step of the estimation procedure.

Demand. Finally, given $\hat{\theta}_S$ and $\hat{\eta}_P := (\hat{\eta}_P^1, \hat{\eta}_P^2, \ldots, \hat{\eta}_P^H)$, I set the technology parameter p_D to minimize the squared distance between the model implied contracts $w_D(p_D; \hat{\theta}_S, \hat{\eta}_P)$ and the empirical wages w_D^{data} used in the first stage,

$$\hat{p}_D = \arg\min_{p_D} (w(p_D; \hat{\theta}_S, \hat{\eta}_P) - w_D^{\text{data}})^2.$$

The main benefit of estimating the supply parameters in the first stage is that it can be done in partial equilibrium. Hence, the full model does not need to be solved for every vector of parameters considered, which would not be feasible. Demand-side estimation does require solving for the full equilibrium, but it is only a one-dimensional problem.

Heuristic Discussion. While the supply side parameters are all jointly estimated, a heuristic discussion about which moments are most informative for each of the parameters is helpful. The conditional choice probabilities (CCPs) determine mobility across the dominant firm, the peripheral firms, and non-employment. Hence, they help pin down home production, the switching costs, the taste shocks, and amenities. The scale of the taste shocks σ is informed by how often workers make an EE transition which results in lower wages. For example, in the top representative market which I define in Section 4.4, these are moves from the dominant firm to any of the peripherals (see Table 9). Hence, $P^1_{D\to P}, P^2_{D\to P}$, and $P^3_{D\to P}$ determine the magnitude of the variance of the taste shocks. The switching cost incurred following an EE transition is pinned down by the rate at which workers switch employers. Consider workers employed at a peripheral firm deciding whether to stay or to go to a rival peripheral firm. The log-odds ratio is given by

$$\ln \frac{P^h_{P\to P}}{P^h_{P\to P'}} = \frac{1}{\sigma}\Big(v(h, P, \chi, P) - v(h, P, \chi, P')\Big) = \frac{\phi^{EE}}{\sigma}$$

where the simplification uses Equation (10). Conditional on knowing σ, this identifies ϕ^{EE}. Next, the amenity values $a_D = (a_D^1, a_D^2, a_D^3)$ are identified from the rate at which workers switch from the dominant firm to any of the peripheral firms, or vice versa. However, the mapping is not as transparent as the one above because of the switching costs which make the worker continuation values origin-specific.[38] As the only feature of the model differentiating firm *types*, i.e. dominant versus peripheral, the amenity values are also pinned down by the relative size of both firm types. All else equal, a big gap between the employment shares of the dominant and peripheral firms will imply large amenity values. Finally, the cost of moving out of non-employment ϕ^{NE} and production while non-employed $b = \{b^1, b^2, b^3\}$ are pinned down by the rates at which workers transition in and out of non-employment. As before, the corresponding log-odds ratio does not provide a transparent mapping to the parameters because human capital accumulates differently for the employed and non-employed workers.[39] Finally, the second target for the supply-side parameters is the law of motion for the aggregate distribution χ, which informs the parameters governing the evolution of human capital $\{g_E^1, g_E^2, g_N^3, g_N^2\}$.

4.3 Discretization

Having described the estimation procedure, I now explain how I compute the moments $P^{\text{data}}, w^{\text{data}}, \chi^{\text{data}}$. This entails inferring non-employment and discretizing the data into $H = 3$ human capital levels.

Non-employment. The DADS datasets are constructed from employment spells. If a worker is not employed, they do not appear in the data. Hence, unemployed workers receiving benefits are not observable. This is why I do not model unemployment and instead focus on non-employment. Nonetheless, to estimate the model, I need to both identify non-employed workers, as opposed to workers that are out of the labor force, and infer which labor market they are attached to. I label a worker as non-employed in a market in year t if I observe them employed in the same market in both years $t-1$ and $t+1$. I then drop workers who

[38] Consider for example the rate at which workers at a peripheral firm decide to stay relative to the rate at which they go to the dominant firm,

$$\ln \frac{P^h_{P\to P}}{P^h_{P\to D}} = \frac{1}{\sigma}\left(w_P^h - w_D^h - a_D^h + \phi^{EE} + \beta(1-\mu)\mathbb{E}_{h'|h,N}V(h', P, \chi') - \beta(1-\mu)\mathbb{E}_{h'|h,E}V(h', D, \chi')\right).$$

Although workers employed at either the dominant or peripheral firm are subject to the same human capital accumulation process, the worker values are not equal because of the switching costs. Hence the continuation values in the expression above do not cancel and we do not have a clear mapping from the moments to a_D^h.

[39] Consider for example the differential rate at which the non-employed remain non-employed relative to moving to a peripheral firm,

$$\ln \frac{P^h_{N\to N}}{P^h_{N\to P}} = \frac{1}{\sigma}\left(b^h - w_P^h - \phi^{NE} + \beta(1-\mu)\mathbb{E}_{h'|h,N}V(h', N, \chi') - \beta(1-\mu)\mathbb{E}_{h'|h,E}V(h', P, \chi')\right).$$

Since the human capital accumulation process differs for employed and non-employed workers, the continuation values do not cancel.

are non-employed for three years or more in a row, which I consider to be out of the labor force and not actively searching for employment.

K-means Clustering. I bin observations of workers across years into $H = 3$ different human capital levels using the k-means clustering algorithm. In the model, because of the taste shocks, there will never be zero mass at any of the points in the joint distribution of workers across human capital levels and firms and non-employment. Thus the main consideration when clustering workers in the data is to do so in a way that does not yield 0 workers in any of the bins. For this reason, I perform the clustering separately for each firm type (leader, non-leader, non-employment) and market tuple. I cluster workers using the following characteristics: employment status, wages, hours, experience, age, and gender.[40]

4.4 Representative Markets

The model introduced in Section 2 is for a single labor market. Since there are tens of thousands of markets, it is not feasible to estimate the model separately for each market in the data. Instead, I segment the data in two according to the employment share of the leading firm: if the employment share of the leader in a market is below (above) the median then I classify the market as belonging to the bottom (top). I make this split so that each of the two markets contains 50% of the workers in France therefore I refer to them as representative markets. The (employment weighted) mean leader share is 20%, with a long right tail (see Table 4 and the y-axis of the heatmap in Appendix A.5 for more). In Table 6, I report summary statistics for each computed in the DADS-Postes.[41] The representative market grouping markets where the leader has a low employment share (henceforth *bottom*) has many firms and workers and is not concentrated as measured by the employment HHI. However, the leader employs roughly ten times more workers than the other firms which suggests a fair amount of market power. The other representative market (henceforth *top*) has a lot fewer firms and workers and a very high employment HHI.

[40] As previously mentioned, I do not observe worker education in the data.

[41] The moments I use for estimation are computed in the DADS-Panel. However, since it only contains $1/12th$ of workers (those born in October), it is not possible to accurately compute employment shares and the employment HHI. Hence I segment markets along the leader's employment share in the DADS-Postes first before computing the moments for estimation in the DADS-Panel.

Table 6: Representative Markets Summary Statistics

	Bottom	**Top**
Number of Firms	2,057	166
Number of Workers	8,472	1,867
Employment HHI	140	2,305
Leader Share (%)	5.4	35.7
Avg. Non-Leader Share (%)	0.46	2.57
Number of Markets	14,959	47,171

Note: Means weighted by market employment. Computed in DADS Postes 2015.

In both representative markets, I compute the (market) employment weighted means of the CCPs P^{data}, wages w^{data}, and the distribution χ^{data} of workers across human capital levels and firms which I report in Section 4.5.

4.5 Model Fit

Supply. The empirical and model implied moments for the supply-side estimation are presented in Table 7 and Table 8. Overall, the model fits the moments well and the fit is slightly better for the top representative market. The model is able to match the CCPs out of the dominant firm and non-employment, with the sole exception being the probability workers at the dominant firm choose non-employment ($P^h_{D\to N}$) in the bottom representative market. However, it misses on the probability workers remain at a peripheral firm ($P^h_{P\to P}$), underestimating these by roughly an order of magnitude. Instead, in the model, more workers go to other peripheral firms ($P^h_{P\to P'}$). Moreover, as before, for the bottom representative market, the model struggles to match the rate at which workers at the peripheral firms choose non-employment ($P^h_{P\to N}$).

Table 7: Model Fit of Conditional Choice Probabilities

	Top									
	Data					**Model**				
		D	P	P'	N		D	P	P'	N
$h = h_1$	D	0.55	0.41		0.05	D	0.57	0.41		0.03
	P	0.13	0.41	0.40	0.05	P	0.14	0.06	0.75	0.05
	N	0.18	0.65		0.17	N	0.14	0.73		0.13
		D	P	P'	N		D	P	P'	N
$h = h_2$	D	0.52	0.43		0.05	D	0.53	0.44		0.03
	P	0.12	0.41	0.41	0.05	P	0.13	0.06	0.76	0.06
	N	0.15	0.67		0.18	N	0.12	0.73		0.15
		D	P	P'	N		D	P	P'	N
$h = h_3$	D	0.54	0.42		0.04	D	0.54	0.42		0.03
	P	0.14	0.41	0.40	0.05	P	0.13	0.06	0.74	0.07
	N	0.20	0.63		0.18	N	0.12	0.70		0.18
	Bottom									
	Data					**Model**				
		D	P	P'	N		D	P	P'	N
$h = h_1$	D	0.33	0.58		0.09	D	0.35	0.65		0.00
	P	0.04	0.37	0.51	0.08	P	0.05	0.02	0.93	0.00
	N	0.05	0.73		0.22	N	0.04	0.74		0.22
		D	P	P'	N		D	P	P'	N
$h = h_2$	D	0.33	0.57		0.10	D	0.33	0.67		0.00
	P	0.04	0.38	0.49	0.09	P	0.05	0.02	0.93	0.00
	N	0.06	0.69		0.26	N	0.03	0.71		0.26
		D	P	P'	N		D	P	P'	N
$h = h_3$	D	0.35	0.57		0.10	D	0.35	0.65		0.00
	P	0.05	0.38	0.47	0.09	P	0.05	0.02	0.92	0.00
	N	0.07	0.67		0.26	N	0.04	0.70		0.26

Note: This table reports both the empirical and estimated conditional choice probabilities for both representative markets. The empirical probabilities are computed in the DADS-Panel.

Next, Table 8 presents the empirical and model implied stationary distribution (in percentages). In both markets, the model fit is almost perfect. The only aspect the model struggles to match is non-employment. In the bottom representative market, the non-employment rate is only half of what it is in the data, and in the top representative market, it is about 50% bigger. The missing mass in non-employment is attributed to the peripheral firms: about 5% more workers and 2% less workers are at peripheral firms in the bottom and top markets, respectively. This discrepancy is tightly linked to the model fit of the CCPs discussed above. For example, in the bottom representative market, workers barely go to non-employment from employment compared to the data which is why the model is unable to match the non-employment rate.

Table 8: Model Fit of Stationary Distribution

	Bottom		**Top**	
	Data	Model	Data	Model
Dominant				
$h = h_1$	2.26	2.22	7.31	7.29
$h = h_2$	2.05	2.03	6.92	6.94
$h = h_3$	2.34	2.32	7.65	7.65
Total	6.65	6.57	21.9	21.9
Non-Employed				
$h = h_1$	3.20	3.73	1.86	4.36
$h = h_2$	3.54	0.91	2.14	2.14
$h = h_3$	3.56	0.34	1.94	1.94
Total	10.3	4.98	5.94	8.45
Total Peripheral				
$h = h_1$	29.6	29.8	23.3	22.0
$h = h_2$	29.9	29.4	24.7	23.7
$h = h_3$	26.5	29.3	24.2	24.0
Total	83.0	88.5	72.2	69.7

Note: This table reports the empirical and estimated distribution across human capital levels, firms, and non-employment. The units are percents.

Demand. Table 9 reports the empirical wages for both the dominant and peripheral firms as well as the model implied dominant wages. For the bottom representative market, the fit is remarkably good. On average, the absolute distance from the empirical wages is 3.3%, with the biggest miss being wages for the most skilled workers which are 5.4% higher than in the data. The model is even able to replicate the non-monotonicity in wages present in the data. For the top representative market, the fit is also very good. The average distance from the empirical wages is 3.9%, with the biggest miss being wages for the most skilled workers which are 5.4% lower than in the data. As before, the model can also replicate the non-monotonicity.

Table 9: Model Fit of Wages

	Bottom			**Top**		
		w_D^h			w_D^h	
	w_P^h	Data	Model	w_P^h	Data	Model
$h = h_1$	21,325	21,662	21,017	22,178	24,768	24,274
$h = h_2$	24,012	17,726	17,960	22,359	28,068	26,754
$h = h_3$	25,693	19,036	20,130	23,433	25,764	24,492
Mean	23,615	19,532	19,744	22,663	26,219	25, 161

Note: This table reports dominant and peripheral wages for both representative markets. The first column in each market is w_P^h, the employment-weighted mean wage offered to workers with human capital h by non-leader firms. The second column is w_D^h, the employment-weighted mean wage offered by the leader. The third column is the model-implied dominant wages at the estimated parameter values. The mean is weighted by the empirical and model-implied aggregate distributions of human capital. The units are 2015 euros.

4.6 Parameter Estimates

Estimates. I now briefly discuss the parameter estimates reported in Table 10 before turning to the welfare losses. Both sets of estimates imply that firms have a significant amount of monopsony power. In the bottom representative market, the amenity value to workers of being employed at the dominant firm relative to a peripheral firm is 3,113, 9,648, and 10,128 euros depending on the worker's human capital level. These correspond to 14%, 54%, and 53% of the wage.[42] In the top representative market, the amenity value estimates are even larger: 120%, 90%, and 114% of wages. Such big amenity values are necessary for the model to match the dominant firm's employment share. The dominant firm is much larger than the peripheral firms and this difference can not fully be explained by the gap in wages which is relatively small. In fact, for some workers, wages at the peripheral firms are higher. Furthermore, the taste shocks do not help either because they differentiate all firms and not specifically the dominant firm versus any of the peripheral firms. The only differentiating factor between firm *types* is the amenities, which the model infers must be very large to explain such a difference in employment shares.

[42] The non-monotonicity is inherited from the wages.

Table 10: Parameter Estimates

Parameter	Description	Bottom	Top
a_D^1	Amenities	3,113	29,674
a_D^2		9,648	25,334
a_D^3		10,128	29,389
ϕ^{EE}	Switching costs	2,958	27,585
ϕ^{NE}		48,006	41,862
σ	Scale of taste shocks	0.13	1.39
b^1	Home production	21,973	32,682
b^2		24,169	35,012
b^3		25,451	38,179
p_D	Productivity gap	2.27	3.99
g_E^1	Learning rates	0.05	0.05
g_E^2		0.03	0.04
g_N^3	Skill decay	0.62	0.14
g_N^2		0.62	0.03

Note: Home production, switching costs, standard deviations of the taste shocks, and amenity values at the dominant firm are in 2015 euros.

Next, the estimates for the switching costs are also very large, giving firms additional market power. In the bottom representative market, the cost of switching firms is roughly 15% of average wages, whereas in the top representative market it is roughly equal to the average wage. This difference stems from two related features of the data: 1. the bottom representative market has a lot more firms than the top representative market, and 2. it has bigger EE rates across all human capital levels and firm types, e.g. 58% of workers at the dominant firm move to a peripheral firm for $h = h_1$ versus 41% in the top representative market. Through the lens of the model, these two features of the data imply that switching firms is much more costly in the top representative market. In contrast, the cost of moving out of non-employment is much larger at approximately twice the average wage in both representative markets. Finally, the last parameter governing the extent of monopsony power is the scale of the taste shocks. The estimates imply a standard deviation of 1,647 euros and 17,883 euros in the bottom and top representative markets, respectively. Hence workers have strong preferences for the different firms and non-employment, above and beyond the differences in wages, home production, and amenities.

For production, I find that the productivity gap between the dominant firm and the peripheral firms is large: 2.27 in the less concentrated markets and 3.99 in the more concentrated markets. Home production of the non-employed is roughly 88% and 65% of the sum of wages and amenities at the dominant firm in the bottom and top representative markets, respectively.[43]

Finally, I find that the skills of workers decay faster than they appreciate. Human capital appreciates relatively slowly while on the job, likely because there are only three different human capital levels. In contrast, workers' skills depreciate an order of magnitude faster when they are non-employed. This makes non-employment particularly costly both for the workers and the firms. A worker with $h = h_3$ in the bottom representative market who becomes non-employed will see their skill decay to $h = h_2$ with probability 62%. Upon re-entry to employment, their human capital appreciates back to $h = h_3$ with probability 5%, meaning that on average it will take them 20 periods (years) to recover. This is not as stark in the top representative market where estimates of g_N^3 and g_N^2 are lower.

5 Welfare Costs of Dynamic Monopsony

Having estimated the model parameters, I now turn to quantifying the welfare costs of monopsony in the presence of on-the-job learning. I first discuss markdowns for both firm types and then quantify the welfare gains from forcing firms to price competitively.

5.1 Markdowns

Peripheral Markdowns. The first six columns of Table 11 present the estimated labor supply elasticities ε_P^h, the markdowns η_P^h, and the gap between wages in the decentralized and centralized equilibria ν_P^h for the peripheral firms. Recall that since the peripheral firms are *static* monopsonists, we have $\eta_P^h = \nu_P^h$. An elasticity of 15.3 means that for a 1% decrease in wages offered, employment decreases by 15.3%. A markdown of 0.939 means that the worker receives 93.9% of their marginal product of labor. A high markdown therefore corresponds to the firm having less wage-setting power. There are two main takeaways from these estimates. First, as expected, the markdowns are lower in the bottom representative market. These correspond to less concentrated markets where there are many firms and the dominant firm is not as big as in the top representative market. Furthermore, the parameter estimates discussed in Section 4.6 imply that firms have less monopsony power, e.g. workers do not have very strong idiosyncratic preferences for the different firms and non-employment.

[43] In terms of market shares, non-employment is closer to the dominant firm than to the peripheral firms.

Therefore if one of the many peripheral firms suddenly offers workers a lower wage, holding all else equal, it will attract fewer workers. In the top representative market, the elasticity is smaller because there are fewer firms and workers have stronger taste shocks. So workers are less sensitive to one-time wage drops. Second, note that within a firm, markdowns are increasing in the human capital of workers. This monotonicity is inherited from the static component of the marginal product of labor, which is also increasing in human capital. Since the peripheral firms behave as if they were atomistic, the wages they offer do not reflect any future changes in human capital.

Table 11: Labor Supply Elasticities and Markdowns

	Peripheral						**Dominant**					
	Bottom			Top			Bottom			Top		
	ε_P^h	η_P^h	ν_P^h	ε_P^h	η_P^h	ν_P^h	ε_D^h	η_D^h	ν_D^h	ε_D^h	η_D^h	ν_D^h
$h = h_1$	15.3	0.939	0.939	1.56	0.610	0.610	13.5	0.931	0.420	1.12	0.528	0.171
$h = h_2$	17.0	0.944	0.944	1.57	0.612	0.612	11.3	0.919	0.307	1.33	0.571	0.192
$h = h_3$	18.1	0.948	0.948	1.65	0.623	0.623	11.8	0.922	0.310	1.19	0.543	0.172
Mean	16.7	0.943	0.943	1.59	0.615	0.615	12.2	0.924	0.348	1.21	0.547	0.178

Note: This table reports, for both estimated markets, both firm types, and for all human capital levels, the labor supply elasticity ε, the markdown η, and the gap ν between the wages offered in the decentralized and centralized equilibria. The mean is weighted by the aggregate distribution of human capital.

Dominant Markdowns. The second six columns of Table 11 report the estimated labor supply elasticities ε_D^h, the markdowns η_D^h, and the gap between wages in the decentralized and centralized equilibria ν_D^h for the dominant firm. Workers are less sensitive to wage changes at the dominant firm: the average labor supply elasticities are roughly 27% and 24% smaller than the peripheral firms' labor supply elasticities in the bottom and top markets, respectively. The reason for this is twofold. First, the amenities are a form of classic differentiation between the dominant and peripheral firms. The estimates for the amenity values are big, i.e. workers have strong preferences for the dominant firm above and beyond the wage differential. Second, there are a lot of peripheral firms. So when one peripheral firm lowers its wages, workers react by not only going to the dominant firm and non-employment but also by going to the other peripheral firms. These lower elasticities imply that the dominant firm can charge a lower markdown, i.e. pay workers a smaller share of their marginal product of labor. As with the peripheral firms, the markdown is higher for the bottom representative market. On average, the dominant firm pays workers about 92% and 55% of their full marginal product (both the static and dynamic components) in the bottom and top representative markets, respectively.

However, as shown in Corollary 2, the gap to the efficient allocation ν_D^h is not equal to the markdown η_D^h. In this case, we have $\nu_D^h < \eta_D^h$ in both markets and for all h. For example, in the bottom representative market for the lowest human capital level wages are 35% of the static marginal product, i.e. the wage that decentralizes the efficient allocation. This is much smaller than the markdown which is about 92%. Such large gaps to the efficient outcome will imply that the welfare gains from forcing firms to price competitively will be very large.

5.2 Welfare

I now look at the welfare implications of forcing firms to offer the wage that decentralizes the efficient allocation, i.e. by setting $\nu_D^h = \nu_P^h = 1$ for all h, which I report in the last columns of Table 12.. Recall that welfare is the planner's objective, given by Equation (24). Note that this jointly forces *all* firms to price competitively, not just the dominant firm or a peripheral firm, so this exercise is equivalent to shutting off all wage-setting power in the market. Moreover, because of the magnitudes of ν_D^h (see Table 11), this exercise entails a roughly three-fold increase in the wages of the dominant firm. I find that the welfare losses are very large in both markets. Welfare and output are approximately 60 percent lower than they would be if $\nu_D^h = \nu_P^h = 1$ for all h. There are also significant human capital costs: the aggregate distribution of human capital is shifted to the left relative, i.e. there are more low-skill workers ($h = h_1, h_2$) and less high-skill workers ($h = h_3$).

The large numbers are to be expected given the estimates presented in the previous section. As Table 11 shows, ν_i^h is low for all i, h and especially so for the dominant firm. Moreover, the dominant firm is not only much more productive than the peripheral firms (p_D is 2.3 and 4 in the bottom and top markets, respectively) but also offers workers higher utility because of the amenity values. Hence, when setting $\nu_i^h = 1$, utility at the dominant firm far eclipses what workers would receive in either employment or non-employment and so they almost all move to the dominant firm. Because of the taste shocks, there are still workers who prefer the peripheral firms or non-employment, but not many.

There are two main sources behind these welfare losses. First, there is an employment effect: increasing wages drives workers out of non-employment. Second, there is a reallocation effect within employment between the two firm types, i.e. workers move from the peripheral firms to the dominant firms. Hence there are more employed workers which means human capital in the market is higher. Moreover, there are more workers at the dominant firm which means output and welfare increase. The increase in output is purely mechanical:

the dominant firm is significantly more productive than the other firms so an increase in employment at the dominant firm will increase market-wide output. The increase in welfare above and beyond output is driven by the amenities and switching costs.

Table 12: Counterfactual Welfare

	Bottom				**Top**			
	10%	25%	50%	100%	10%	25%	50%	100%
Leader Share	–93%	–93%	–93%	–93%	–66%	–76%	–77%	–77%
Non-employment Rate	+21%	+27%	+27%	+27%	+74%	+196%	+232%	+236%
Share $h = h_1$	+1%	+1%	+1%	+1%	+4%	+6%	+6%	+6%
Share $h = h_2$	–1%	–1%	–1%	–1%	+10%	+17%	+18%	+18%
Share $h = h_3$	–1%	–1%	–1%	–1%	–12%	–18%	–18%	–19%
Output	–51%	–52%	–52%	–52%	–44%	–57%	–58%	–58%
Welfare	–58%	–59%	–59%	–59%	–54%	–66%	–68%	–68%

Note: This table reports counterfactual outcomes when the wages get closer to the ones that decentralize the efficient allocation: $w_i^h + x \times (w_{i,S}^h - w_i^h)$ for $x \in \{0.1, 0.25, 0.5, 1\}$. Units are percent change of the decentralized equilibrium relative to the counterfactual economy considered.

This counterfactual is perhaps not very pragmatic since no policy instrument forces firms to price competitively. However, policies like the minimum wage may allow us to partially bridge the gap. In the first three columns for each representative market in Table 12, I report the effects of increasing wages 10, 25, and 50 percent closer to the efficient wages. For the bottom representative market, the benefits are almost fully reached when bridging the gap by only 10% (and fully reached for 25% and 50%). This is because the labor supply elasticities are big and so a 10% increase is enough to push all of the workers to the dominant firm. This is a somewhat striking result that may be of particular interest to policymakers: in the large markets with lower concentration, only (relatively) small changes in wages suffice to recover almost the entirety of the welfare gains from removing monopsony power. For the top representative market, the welfare losses are more progressive as the gap in wages is bridged. Nonetheless, even the relatively small increase of 10% results in about 80% of the total output and welfare losses possible, and about 2/3 of the losses in human capital.

5.3 Decomposing the Welfare Losses

Finally, I examine how much of the welfare losses can be attributed to the dynamics. The only model ingredient making the environment dynamic is human capital accumulation.

Hence, isolating the static effects amounts to removing changes in human capital. To do so, I remove the birth/death process and learning by setting $\mu = g_E^1 = g_E^2 = g_N^3 = g_N^2 = 0$. Workers are now infinitely-lived and their human capital is immutable. I hold all other supply-side parameters fixed at their estimates reported in Table 10. Hence, the labor supply elasticities and markdowns are also the same as the ones reported in Table 11. However, the estimates for the productivity gap p_D are no longer the same. Without the dynamics, the wage equation for the dominant firm, Equation (16), reduces to

$$w_D^h = \frac{\varepsilon_D^h(\chi, w_D)}{\varepsilon_D^h(\chi, w_D) + 1} \times \frac{\partial F_D(\chi, w_D)}{\partial m_D^h(\chi, w_D)},$$

which can be inverted to recover p_D. The dominant firm is now also a static monopsonist like the peripheral firms. Nonetheless, it remains differentiated because of the amenities. So we should still expect to find $p_D > 1$. The new estimates for p_D are much smaller than they were with dynamics: 1.02 (compared to 2.27 previously) and 1.34 (compared to 3.99 previously) for the bottom and top representative markets, respectively.[44] Note that this also highlights the importance of dynamics in estimating p_D. Ignoring human capital accumulation and applying the static Lerner formula above leads to estimates of p_D that are biased downwards.

Table 13: Decomposition of the Welfare Losses

	Bottom		**Top**	
	No dynamics	Full	No dynamics	Full
Leader Share	–90%	–93%	–65%	–77%
Share $h = h_1$	+0%	+1%	+0%	+6%
Share $h = h_2$	+0%	–1%	+0%	+18%
Share $h = h_3$	+0%	–1%	+0%	–19%
Output	+6.8%	–52%	+37%	–58%
Welfare	–18%	–59%	–4.5%	–68%

Note: The first columns report counterfactual outcomes when shutting off human capital accumulation. The second columns are copied from Table 12. Units are percent change of the decentralized equilibrium relative to the counterfactual economy considered.

In Table 13, I report the welfare losses for this counterfactual where the dynamics are turned off. I hold fixed the aggregate distribution of human capital. Without the birth/death

[44] There are H estimates of p_D when inverting this equation. They are almost identical, but to be conservative I use the largest value.

process, the non-employment rate is machine epsilon[45], so I do not report the change. Output is lower in the centralized equilibrium with no human capital accumulation than in the decentralized equilibrium because of the difference in p_D estimates. The welfare losses are underestimated by 69 and 93 percent in the bottom and top markets, respectively, when failing to account for human capital accumulation.

6 Conclusion

In this paper, I study how the welfare costs of monopsony change when accounting for endogenous human capital accumulation. In doing so, I make several contributions to the existing literature. Motivated by novel descriptive evidence on local labor markets in France and their competitive structure, I develop a general equilibrium model of segmented markets. The model features worker heterogeneity in the form of general human capital and a finite number of firms that compete for them. Importantly, the model contains rich dynamics due to human capital accumulation. I leverage tools from the machine learning literature on deep reinforcement learning to solve the equilibrium. I then study a planner who maximizes market-wide welfare by allocating workers across firms and non-employment and characterize the wages that decentralize the efficient outcome. I find that the markdown, which now has an additional dynamic term relative to its usual formulation, is no longer a sufficient statistic for the welfare losses due to monopsony. I estimate the model using rich administrative matched employee-employer data from France and show that the welfare gains from forcing firms to price competitively, or even just a bit closer to competitively, are large both in terms of output and human capital.

I believe this paper provides several avenues for future research. Perhaps the most promising is to study how the interaction of on-the-job learning and monopsony power changes the policy implications of monopsony. In future work, I plan to modify the framework to examine the effects of the minimum wage and mergers. The model is particularly well suited to studying mergers because there are finitely many firms. Another potentially fruitful extension would be to allow for richer human capital dynamics, both in terms of adding an intensive margin and firm-specific human capital. As discussed in Section 2.8, the framework can accommodate these changes. Finally, I believe this paper showcases the importance of adding dynamics to workhorse models of monopsony. Human capital accumulation is one of many ways of doing this and more work is needed to fully understand what shapes firms' incentives to price below the marginal product of labor.

[45] It is not exactly 0 because of the taste shocks, but the planner reallocates almost all the workers to employment.

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Appendix

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A Data Appendix

A.1 Variables

- Wages: I use the gross annual wage, measured by the variable `S_BRUT` in the DADS-Postes and the variable `SB` in the DADS-Panel. These are neither top- nor bottom-coded and include base salary, overtime, and bonuses. I deflate wages to 2015 euros throughout using the CPI provided by the INSEE at https://www.insee.fr/fr/statistiques/2122401#tableau-figure1.

- Hours: I use reported yearly hours worked as measured by the variable `NBHEUR`. As discussed in the main text, it is not possible to distinguish between reported hours and actual hours.

- Commuting Zone: I use the commuting zone of the establishment given by the variable `ZEMPT` in DADS-Postes. I use the crosswalk[46] between regions, communes, and commuting zones provided by the INSEE to infer commuting zones in the DADS-Panel.

- Occupation: I use 4-digit occupation codes. The full list of the 360 distinct occupation codes can be found on INSEE's website at https://www.insee.fr/fr/metadonnees/pcs2003/?champRecherche=true.

A.2 Herfindahl-Hirschman Index

The employment Herfindahl-Hirschman Index (HHI) for market i is defined as $\text{HHI}_i := \sum_{j \in i} s_j^2$, where

$$s_j := \frac{e_j}{\sum_{k \in i} e_k} \times 100$$

is the employment share of firm j. Note that the square gives proportionally more weight to firms with bigger market shares. The largest HHI value is 10,000 which corresponds to a market with a single firm. The Horizontal Merger Guidelines (DOJ and FTC 2010) classifies markets as follows:

- HHI $\leq$ 1,500: unconcentrated
- HHI $\in (1,500, 2,500)$: moderately concentrated
- HHI $\geq$ 2,500: highly concentrated

The HHI can also be computed using wage shares instead of employment shares.

[46] The crosswalk can be downloaded at https://www.insee.fr/fr/information/2028028.

A.3 Occupations

Table A.2: Occupation Summary Statistics

	Mean	SD	p25	Median	p75	p95
Workers/ 2-digit occ	567,448	573,765	32,692	333,143	1,122,509	1,751,103
Workers/ 4-digit occ	42,559	62,524	8,550	22,585	44,631	174,875
2-digit occ/firm (unweighted)	2.18	1.93	1	1	3	6
2-digit occ/firm (weighted)	3.66	6.58	1	1	1	22
4-digit occ/firm (unweighted)	3.10	5.29	1	2	3	10
4-digit occ/firm (weighted)	27.6	67.6	3	3	3	243
Number of 2-Digit Occupations	27					
Number of 4-Digit Occupations	360					

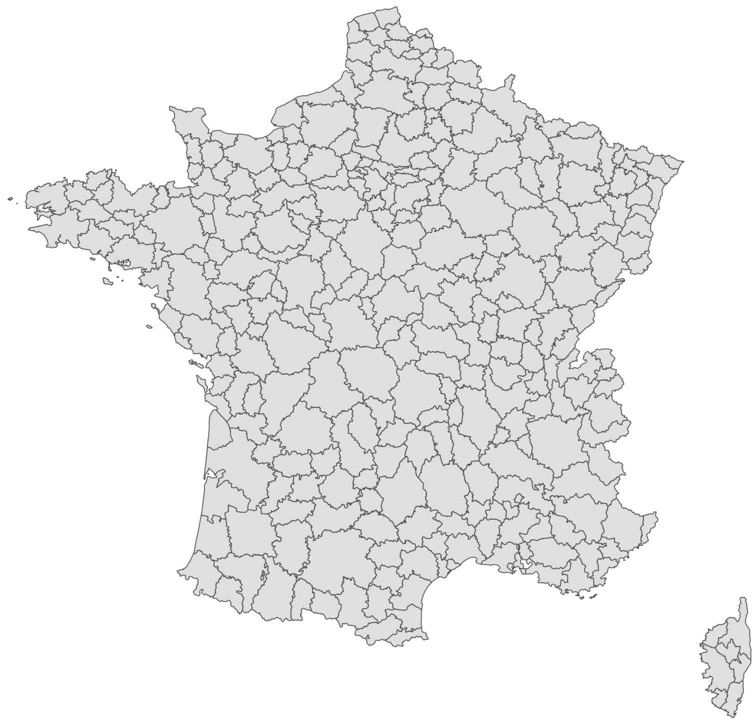
Note: Weighted by firm employment.

Table A.1: Granularity of PCS-ESE occupation codes

4		**Intermediate Professions**
42		Schoolteachers, instructors and similar
43		Intermediate health and social work professions
	431a	Nursing executives
	431b	Psychiatric nurses
	431c	Childcare workers
	431d	Specialized nurses (other than psychiatric and pediatric)
	431e	Midwives (salaried)
	431f	Nurses in general care (salaried)
	432b	Rehabilitation massage therapists (salaried)
	432d	Other rehabilitation specialists (salaried)
	433a	Medical technicians
	433b	Eyewear opticians and audioprosthetists (salaried)
	433c	Other medical device specialists (salaried)
	433d	Pharmacy assistants
	434a	Socio-educational intervention managers
	434b	Social work assistants
	434c	Family counselors
	434d	Specialized educators
	434e	Support workers
	434f	Specialized technical educators, workshop instructors
	434g	Early childhood educators
	435a	Directors of socio-cultural and leisure centers
	435b	Socio-cultural and leisure organizers
44		Clergy, religious
45		Intermediate administrative professions of the public service
46		Intermediate administrative and commercial professions of companies
47		Technicians (except service technicians)
48		Foremen and supervisors

Note: This table illustrates occupation coding in the DADS, the PCS-ESE. Intermediate Professions (1-digit occupation code 4) is split into 7 2-digit occupation codes. Each of these is further split into many 4-digit occupation codes. This split is shown for Intermediate health and social work professions. The full list of occupation codes can be found on INSEE's website at `https://www.insee.fr/fr/metadonnees/pcs2003/?champRecherche=true`.

A.4 Commuting Zones



Note: Map of the 304 metropolitan commuting zones of France according to the 2010 zoning rules. In addition to these, there are 18 overseas commuting zones in Guadeloupe, Martinique, Guyane, La Réunion, and Mayotte. The map with the shapefile created by Opendatasoft using geographic data from the French government at
https://www.data.gouv.fr/fr/datasets/contours-des-regions-francaises-sur-openstreetmap/.

Figure A.1: Metropolitan Commuting Zones of France

Table A.3: Commuting Zones Summary Statistics

	Mean	SD	p25	Median	p75	p95
Workers/CZ	51,212	148,204	11,208	20,150	47,628	155,161
Firms/CZ (weighted)	139,420	69,862	43,420	180,217	180,217	180,217
4-digit occ/CZ (weighted)	355	39	360	360	360	360
2-digit occ/CZ (weighted)	27	3	27	27	27	27
Number of CZs	305					

Note: Weighted by commuting zone employment.

A.5 Additional Heatmap

Note: This plots the joint distribution of the ratio of the leader's employment share and the market size (number of firms). Both variables are employment-weighted and split into deciles so that each row and column contains 10% of workers.

Figure A.2: Heatmap between Leader Share and Market Size (Deciles)

A.6 Alternative Market Definition

Table A.4: Market Summary Statistics (2-digit occ × CZ)

	Mean	SD	p25	Median	p75	p95
Number of Firms (unweighted)	403	1,485	37	122	359	1,490
Number of Firms (weighted)	6,982	14,001	528	1,407	4,523	45,584
Number of Workers (unweighted)	2,001	9,490	74	450	1,620	7,492
Number of Workers (weighted)	47,003	101,583	2,915	7,736	24,234	291,672
Employment HHI (unweighted)	927	1,688	161	373	894	3,456
Employment HHI (weighted)	345	562	38	119	333	1,658
Number of Markets	7,656					

Note: Unweighted statistics computed with equal weights for each market. Weighted statistics use market employment as weights.

Table A.5: Leader Summary Statistics (2-digit occ $\times$ CZ)

	Mean	SD	p25	Median	p75	p95
Leader Share (unweighted)	15	14	6.0	11	20	46
Leader Share (weighted)	11	12	2.8	5.9	13	40
Second Share (unweighted)	7.8	6.8	3.6	6.1	9.8	19
Second Share (weighted)	4.0	3.2	1.7	3.1	5.3	10
Leader Share / Second Share (unweighted)	2.5	4.3	1.2	1.5	2.1	6.8
Leader Share / Second Share (weighted)	4.3	8.9	1.1	1.4	2.3	22
Mean Non-Leader Share (unweighted)	2.0	3.9	0.2	0.7	2.0	8.3
Mean Non-Leader Share (weighted)	0.2	0.4	0.02	0.06	0.1	0.6
Leader Share / Mean Non-Leader Share (unweighted)	39	140	4.3	14	34	121
Leader Share / Mean Non-Leader Share (weighted)	329	647	38	82	285	1,320

Note: Unweighted statistics computed with equal weights for each market. Weighted statistics use market employment as weights. Markets with only one firm (leader's employment share = 100%) are excluded.

B Model Appendix

B.1 Model Notation

Table B.1: Model Notation

Symbol	Description
$H \in \mathbb{N}$	number of human capital levels
$\mathcal{H} := \{h_1, \dots, h_H\}$	human capital grid
$h \in \mathcal{H}$	index of human capital
$\mu \in (0, 1)$	birth/death rate
$M \in \mathbb{N}$	number of peripheral firms
$\mathcal{P} := \{y_1, y_2, \dots, y_M\}$	the set of peripheral firms
$i \in \{D, P\}$	index of firm type
p_i	firm productivity, with $p_D \geq p_P = 1$
$w_i := \{w_i^1, w_i^2, \dots, w_i^H\}$	wage menu chosen by i
$y \in \mathcal{P}$	index of peripheral firm
$\epsilon := \{\epsilon_D, \epsilon_{y_1}, \epsilon_{y_1}, \dots, \epsilon_{y_M}, \epsilon_N\}$	preference shocks for each firm and non-employment
$\sigma > 0$	scale of T1EV preference shocks
$a_D := (a_D^1, a_D^2, \dots, a_D^H)$	amenity value at dominant relative to peripheral
$\phi^{EE}, \phi^{NE} > 0$	switching cost incurred after EE and NE transitions, respectively
$m_i := \{m_i^1, m_i^2, \dots, m_i^H\}$	labor supply curve of i given choice (w_i, n_i)
$F_i(m_i)$	output of firm i with workforce m_i
b^h	home production
g_E^h	prob. human capital of an employed worker appreciates
g_N^h	prob. human capital of non-employed worker depreciates
$(e_P^{y_1}, \dots, e_P^{y_M}, e_D, u) \subset \Delta^{(M+2)\times H}$	aggregate distribution of workers across firms and non-employment
$\hat{e_D}, \hat{e_P}, \hat{u}$	distributions after human capital accumulation
μ	birth/death rate
$s := (s^1, s^2, \dots, s^H)$	total employment at peripheral firms
$\chi := (e_D, u, s) \subset \Delta^{3H}$	aggregate state
$o \in \{D, P, N\}$	worker origin
$d \in \{D, P, P', N\}$	worker destination
$V(h, o, \chi)$	value to a worker with state (h, o, χ)
$u(h, o, \chi, d)$	contemporaneous utility to a worker with state (h, o, χ) choosing d
$v(h, o, \chi, d)$	choice-specific worker value
γ^{EM}	Euler-Mascheroni constant
$P_{o\to d}^h = P(d \vert h, o, \chi)$	conditional choice probability
ε_i^h	labor supply elasticity
η_i^h	markdown
$J_D(\chi)$	value of dominant firm with state χ
$S(\chi)$	planner's value
$S(h, o, \chi)$	planner's integrated value for worker of type (h, o)
ν_i^h	gap between decentralized and centralized wages

B.2 Worker Value

The closed-form expressions for the worker value functions are

$$V(h,D,\chi)=\sigma\Bigg(\ln\Bigg(\exp\frac{v(h,D,\chi,D)}{\sigma}+M\exp\frac{v(h,D,\chi,P)}{\sigma}+\exp\frac{v(h,D,\chi,N)}{\sigma}\Bigg)+\gamma^{EM}\Bigg)$$
$$V(h,N,\chi)=\sigma\Bigg(\ln\Bigg(\exp\frac{v(h,N,\chi,D)}{\sigma}+M\exp\frac{v(h,N,\chi,P)}{\sigma}+\exp\frac{v(h,N,\chi,N)}{\sigma}\Bigg)+\gamma^{EM}\Bigg)$$
$$V(h,P,\chi)=\sigma\Bigg(\ln\Bigg(\exp\frac{v(h,P,\chi,P)}{\sigma}+(M-1)\exp\frac{v(h,P,\chi,P')}{\sigma}+\exp\frac{v(h,P,\chi,D)}{\sigma}+\exp\frac{v(h,P,\chi,N)}{\sigma}\Bigg)+\gamma^{EM}\Bigg)$$

where $\gamma^{EM}\approx 0.5772$ is the Euler-Mascheroni constant and

$$v(h,o,\chi,d)=u(h,o,\chi,d)+\beta(1-\mu)\mathbb{E}_{h'|h,d}V_\sigma(h',d,\chi').$$

B.3 Conditional Choice Probabilities

A worker employed at a peripheral firm can choose to either stay at the *same* peripheral firm, move to another peripheral firm, move to the dominant firm, or go to non-employment. The corresponding choice probabilities are

$$P^h_{P\to P}=\frac{e^{v(h,P,\chi,P)/\sigma}}{e^{v(h,P,\chi,P)/\sigma}+e^{v(h,P,\chi,D)-\phi^{EE}/\sigma}+(M-1)e^{v(h,P,\chi,P')-\phi^{EE}/\sigma}+e^{v(h,P,\chi,N)/\sigma}}$$

$$P^h_{P\to D}=\frac{e^{v(h,P,\chi,D)-\phi^{EE}/\sigma}}{e^{v(h,P,\chi,P)/\sigma}+e^{v(h,P,\chi,D)-\phi^{EE}/\sigma}+(M-1)e^{v(h,P,\chi,P')-\phi^{EE}/\sigma}+e^{v(h,P,\chi,N)/\sigma}}$$

$$P^h_{P\to P'}=\frac{e^{v(h,P,\chi,P)-\phi^{EE}/\sigma}}{e^{v(h,P,\chi,P)/\sigma}+e^{v(h,P,\chi,D)-\phi^{EE}/\sigma}+(M-1)e^{v(h,P,\chi,P')-\phi^{EE}/\sigma}+e^{v(h,P,\chi,N)/\sigma}}$$

$$P^h_{P\to N}=\frac{e^{v(h,P,\chi,N)/\sigma}}{e^{v(h,P,\chi,P)/\sigma}+e^{v(h,P,\chi,D)-\phi^{EE}/\sigma}+(M-1)e^{v(h,P,\chi,P')-\phi^{EE}/\sigma}+e^{v(h,P,\chi,N)/\sigma}}$$

where

$$P^h_{P\to P}+P^h_{P\to D}+(M-1)P^h_{P\to P'}+P^h_{P\to N}=1.$$

A worker employed at the dominant firm can choose to either stay at the dominant firm, move to a peripheral firm, or go to non-employment. The corresponding choice probabilities are

$$P^h_{D\to D}=\frac{e^{v(h,D,\chi,D)/\sigma}}{e^{v(h,D,\chi,D)/\sigma}+Me^{v(h,D,\chi,P)-\phi^{EE}/\sigma}+e^{v(h,D,\chi,N)/\sigma}}$$

$$P^h_{D\to P} = \frac{e^{v(h,D,\chi,P)-\phi^{EE}/\sigma}}{e^{v(h,D,\chi,D)/\sigma} + Me^{v(h,D,\chi,P)-\phi^{EE}/\sigma} + e^{v(h,D,\chi,N)/\sigma}}$$

$$P^h_{D\to N} = \frac{e^{v(h,D,\chi,N)/\sigma}}{e^{v(h,D,\chi,D)/\sigma} + Me^{v(h,D,\chi,P)-\phi^{EE}/\sigma} + e^{v(h,D,\chi,N)/\sigma}}$$

where

$$P^h_{D\to D} + MP^h_{D\to P} + P^h_{D\to N} = 1.$$

Similarly, a worker in non-employment can choose to go to either the dominant firm or the peripheral firms or to stay non-employed:

$$P^h_{N\to D} = \frac{e^{v(h,N,\chi,D)-\phi^{NE}/\sigma}}{e^{v(h,N,\chi,D)-\phi^{NE}/\sigma} + Me^{v(h,N,\chi,P)/\sigma} + e^{v(h,N,\chi,N)/\sigma}}$$

$$P^h_{N\to P} = \frac{e^{v(h,N,\chi,P)-\phi^{NE}/\sigma}}{e^{v(h,N,\chi,D)-\phi^{NE}/\sigma} + Me^{v(h,N,\chi,P)-\phi^{NE}/\sigma} + e^{v(h,N,\chi,N)/\sigma}}$$

$$P^h_{N\to N} = \frac{e^{v(h,N,\chi,N)/\sigma}}{e^{v(h,N,\chi,D)-\phi^{NE}/\sigma} + Me^{v(h,N,\chi,P)-\phi^{NE}/\sigma} + e^{v(h,N,\chi,N)/\sigma}}$$

Again, we have

$$P^h_{N\to D} + MP^h_{N\to P} + P^h_{N\to N} = 1.$$

As in Aguirregabiria and Mira (2002), we can write the worker value in terms of the probabilities:

$$V_\sigma(h,o,\chi) = \sum_{d\in\{D,P,N\}} P^h_{o\to d} \times \left[u(h,o,\chi,d) + \mathbb{E}[\epsilon_d|h,o,\chi,d] + \beta(1-\mu)\mathbb{E}_{h'|h,d}V_\sigma(h',d,\chi')\right]$$

for $o \in \{D,N\}$ and

$$V_\sigma(h,o,\chi) = \sum_{d\in\{D,P,P',N\}} P^h_{o\to d} \times \left[u(h,o,\chi,d) + \mathbb{E}[\epsilon_d|h,o,\chi,d] + \beta(1-\mu)\mathbb{E}_{h'|h,d}V_\sigma(h',d,\chi')\right]$$

for $o = P$.

B.4 Dominant Firm FOCs

The first-order conditions for the dominant firm's choice of wages are

$$(w_D^h) \quad \frac{\partial F_D(m_D(\chi, w_D))}{\partial w_D^h} - m_D(\chi, w_D) - \frac{\partial m_D^h(\chi, w_D)}{\partial w_D^h} w_D^h + \beta \frac{\partial J_D(\chi')}{\partial w_D^h} = 0 \tag{30}$$

Changing wages affects output, the labor supply curves, and the distribution next period. For output, we have

$$\frac{\partial F_D(m_D(\chi, w_D))}{\partial w_D^h} = \sum_h \frac{\partial F_D}{\partial m_D^h} \frac{\partial m_D^h}{\partial w_D^h}$$

which, given the functional form assumption on output (Equation (2)),

$$F_D(m_D) = p_D \sum_{h \in \mathcal{H}} m_D^h \times h,$$

can be simplified to

$$\frac{\partial F_D}{\partial m_D^h} = p_D h.$$

This is the marginal product of labor: $\partial F_D / \partial m_D^h$ is how much output changes when the firm has one more worker with human capital h. I discuss how the firm's choice of policies affects its own labor supply curve through the choice probabilities and next period's distribution in Appendix B.5.

Multiplying Equation (30) by $\frac{\partial m_D(\chi,w_D)}{\partial m_D(\chi,w_D)} \frac{w_D^h}{w_D^h} \frac{m_D(\chi,w_D)}{m_D(\chi,w_D)} \frac{w_D^h}{m_D(\chi,w_D)}$, rearranging, and using the definition of the labor supply elasticity with respect to wages given by Equation (11) yields Equation (16) in the main text

$$w_D^h = \frac{\varepsilon_{D,h}^w(\chi, w_D)}{\varepsilon_{D,h}^w(\chi, w_D) + 1} \left[\frac{\partial F_D(\chi, w_D)}{\partial m_D(\chi, w_D)} + \beta \mathbb{E} \frac{\partial J_D(\chi')}{\partial m_D^h(\chi, w_D)} \right].$$

B.5 Mechanics of non-atomistic dominant firm

In this section, I illustrate what it means for the dominant firm to be non-atomistic. In choosing wages, the dominant firm fully internalizes its effect on the sorting of workers, not only to itself but also to rival peripheral firms and non-employment. This works through the choice probabilities and shows up in two related ways in the firm's first-order conditions presented in Appendix B.4. First, the number of workers that decide to come changes as the firm varies its wages. Differentiating Equation (12), this change is given by

$$\frac{\partial m_D^h(\chi, w_D, n_D)}{\partial w_D^h} = e_D^h \frac{\partial P_{D\to D}^h}{\partial w_D^h} + s^h \frac{\partial P_{P\to D}^h}{\partial w_D^h} + n^h \frac{\partial P_{N\to D}^h}{\partial w_D^h}.$$

Second, the policy choice today affects the continuation value through the distribution the firm will face tomorrow. This effect can be computed using the chain rule:

$$\frac{\partial J_D(\chi_D')}{\partial w_D^h} = \sum_h \left[\frac{\partial J_D(\chi_D')}{\partial (e_D^h)'} \frac{\partial (e_D^h)'}{\partial w_D^h} + \frac{\partial J_D(\chi_D')}{\partial (s^h)'} \frac{\partial (s^h)'}{\partial w_D^h} + \frac{\partial J_D(\chi_D')}{\partial (n^h)'} \frac{\partial (n^h)'}{\partial w_D^h} \right].$$

The first terms can all be computed using the envelope theorem. The dominant firm's value next period is

$$J_D(\chi') = \max_{w_D' \in \mathbb{R}_+^H} p_i \sum_{h\in\mathcal{H}} (m_D^h)' h - \sum_h (m_D^h)'(w_D^h)' + \beta \mathbb{E} J_D(\chi_D'')$$

where

$$(m_D^h)' = (e_D^h)'(P_{D\to D}^h)' + (s^h)'(P_{P\to D}^h)' + (n^h)'(P_{N\to D}^h)'$$

and χ_D'' evolves according to the laws of motion presented in the Section 2.2. So, for example, the change in the value next period given a change in the number of workers with human capital h at the dominant firm next period is

$$\frac{\partial J_D(\chi_D')}{\partial (e_D^h)'} = p_i (P_{D\to D}^h)' h - (P_{D\to D}^h)'(w_D^h)'.$$

The rest of the terms can also be computed using the laws of motions. Again, this all works through the conditional choice probabilities.

I now show how firms affect worker mobility through the conditional choice probabilities. The choice of wages affects all choice probabilities. For ease of exposition, I only report how $P_{D\to D}^h$ changes when the dominant firm raises wages. This can easily be computed as

$$\begin{aligned}
\frac{\partial P_{D\to D}^h}{\partial w_D^h} &= \frac{e^{v(h,D,\chi,D)/\sigma} \frac{\partial v(h,D,\chi,D)}{\partial w_D^h}}{\sigma\left(e^{v(h,D,\chi,D)/\sigma} + M e^{v(h,D,\chi,P)/\sigma} + e^{v(h,D,\chi,N)/\sigma}\right)} \\
&\quad - \frac{e^{v(h,D,\chi,D)/\sigma}\left(e^{v(h,D,\chi,D)/\sigma} \frac{\partial v(h,D,\chi,D)}{\partial w_D^h} + M e^{v(h,D,\chi,P)/\sigma} \frac{\partial v(h,D,\chi,P)}{\partial w_D^h} + e^{v(h,D,\chi,N)/\sigma} \frac{\partial v(h,D,\chi,N)}{\partial w_D^h}\right)}{\sigma\left(e^{v(h,D,\chi,D)/\sigma} + M e^{v(h,D,\chi,P)/\sigma} + e^{v(h,D,\chi,N)/\sigma}\right)^2} \\
&= \frac{1}{\sigma} P_{D\to D}^h \left[(1 - P_{D\to D}^h) \frac{\partial v(h,D,\chi,D)}{\partial w_D^h} - M P_{D\to P}^h \frac{\partial v(h,D,\chi,P)}{\partial w_D^h} - P_{D\to N}^h \frac{\partial v(h,D,\chi,N)}{\partial w_D^h} \right]
\end{aligned}$$

which takes on the usual form (e.g. Train 2009 section 3.6). However, there is one key feature here that separates this from standard dynamic discrete choice models: the supply (usually demand in IO models) is dynamic. Indeed, recall that the choice-specific worker value in this case is

$$v(h, D, \chi, D) = w_D^h + a_D^h + \beta(1-\mu)\mathbb{E}_{h'|h,D} V_\sigma(h', D, \chi'_D).$$

The presence of a continuation value makes the worker's decision a dynamic one. This added component implies that $\frac{\partial P^h_{D\to D}}{\partial w_D^h}$ is a forward-looking expression. To see this, note that

$$\frac{\partial v(h, D, \chi, D)}{\partial w_D^h} = 1 + \beta(1-\mu)\left[\mathbb{1}_{h<h_H} g_E^h \frac{\partial V(h+1, D, \chi')}{\partial w_D^h} + (1-g_E^h)\frac{\partial V(h, D, \chi')}{\partial w_D^h}\right]$$

where $V(h, D, \chi')$ is the worker value next period,

$$V(h, D, \chi') = \sigma\left(\ln\left(\exp\frac{v(h, D, \chi', D)}{\sigma} + M\exp\frac{v(h, D, \chi', P)}{\sigma} + \exp\frac{v(h, D, \chi', N)}{\sigma}\right) + \gamma^{EM}\right).$$

Again, because firms are non-atomistic, the choice of policies today affects the worker value next period through the change in the distribution $\chi' = (e'_D, n', s')$. Applying the chain rule to the worker value like we did to the firm value above,

$$\frac{\partial V(h, D, \chi')}{\partial w_D^h} = \sum_{\hat{h}}\left[\frac{\partial V(h, D, \chi')}{\partial (e_D^{\hat{h}})'}\frac{\partial (e_D^{\hat{h}})'}{\partial w_D^h} + \frac{\partial V(h, D, \chi')}{\partial (s^{\hat{h}})'}\frac{\partial (s^{\hat{h}})'}{\partial w_D^h} + \frac{\partial V(h, D, \chi')}{\partial (n^{\hat{h}})'}\frac{\partial (n^{\hat{h}})'}{\partial w_D^h}\right].$$

Clearly, $\partial V(h, D, \chi')/\partial w_D^h$ is a function of $\partial v(h, D, \chi', D)/\partial w_D^h$, which is itself a function of $\partial V(h, D, \chi'')/\partial w_D^h$, and so on and so forth.

B.6 Non-concavity

I show that the dominant firm's profits are neither concave nor quasi-concave by counterexample. I illustrate this by using the version of the model (no dynamics) that most closely resembles the so-called "logit profit" models in the Operations Research literature (e.g. Dong, Kouvelis, and Tian 2009, Gallego and Wang 2014, and Song, Song, and Shen 2021)[47] which have long been known to be non-concave (Hanson and Martin 1996).

[47] An important result from this literature is that, although the profits are not concave in prices, they are concave in the market shares (conditional choice probabilities). Since there is a one-to-one mapping between prices and market shares, the firm's problem can be recast in probability space. This is a very strong and elegant result that can be used to bypass numerical issues arising from the non-concavity. I have used this to solve the static version of the model by recasting the problem in probability space, but it is not clear if this approach still holds when adding dynamics.

Lemma 1 (Concavity)**.** *The firm value function given by Equation* (15) *is neither concave nor quasi-concave.*

Proof. By counterexample. Suppose $N = 1$ and $H = 2$, i.e. there is just one peripheral firm and two human capital levels low L and high H. Furthermore, consider the following parameterization:

$$(\beta = 0, b^L = 0, b^H = 0, a_D^L = 0, a_D^H = 0, \phi^{EE} = 0, \phi^{NE} = 0, \sigma = 1, p_D = 100).$$

Now, suppose the peripheral firm has chosen wages (equal to human capital)

$$w_P := (w_{P,L}, w_{P,H}) = (23, 31).$$

Next, consider the following two wage schedules for the dominant firm

$$w_D^1 := (w_{D,L}^1, w_{D,H}^1) = (17, 95)$$

$$w_D^2 := (w_{D,L}^2, w_{D,H}^2) = (38, 42)$$

Since there are no switching costs, worker origin does not matter so the firm's labor supply curve is the product of the aggregate distribution of human capital and the conditional choice probability. Suppose half of the workers have low human capital. Then the dominant firm's profits for both of these earnings schedules given w_P are

$$J_D(w_D^1) = 100 - \frac{1}{2}\frac{e^{17}}{e^{17} + e^{23} + e^0} \times 17 - \frac{1}{2}\frac{e^{95}}{e^{95} + e^{31} + e^0} \times 95 \approx 52.5$$
$$J_D(w_D^2) = 100 - \frac{1}{2}\frac{e^{38}}{e^{38} + e^{23} + e^0} \times 38 - \frac{1}{2}\frac{e^{42}}{e^{42} + e^{31} + e^0} \times 42 \approx 60.0$$

Now consider a convex combination of these earnings

$$w_D^3 := \frac{1}{2} \times w_D^1 + \frac{1}{2} \times w_D^2 = (27.5, 68.5)$$

These yield profits

$$\begin{aligned} J_D(w_D^3) &= 100 - \frac{1}{2}\frac{e^{27.5}}{e^{27.5} + e^{23} + e^0} \times 27.5 - \frac{1}{2}\frac{e^{68.5}}{e^{68.5} + e^{31} + e^0} \times 68.5 \approx 52.2 \\ &< \frac{1}{2}J_D(w_D^1) + \frac{1}{2}J_D(w_D^2) \approx 56.2 \end{aligned}$$

so the dominant firm's profits are not concave. We also have

$$J_D(w_D^3) \approx 52.2 < \min\{J_D(w_D^1), J_D(w_D^2)\} \approx 52.5$$

so the dominant firm's profits are not quasi-concave. □

C Computational Appendix

C.1 Fixed point inside the firm problem

The dominant firm's choice of w_D affects its value in three ways:

1. It affects contemporaneous revenue through output and the wage bill.

2. It affects the labor supply m_D through the choice probabilities. All $10 \times H$ probabilities are affected. In Appendix B.5, I illustrate this with $P^h_{D \to D}$, the probability the dominant firm retains a worker of type h. When the dominant firm varies the wage it offers, it varies $P^h_{D \to D}$ in two ways. First, it changes the choice-specific value function $v(h, D, \chi)$. Second, it changes χ' (see point 3. below) and so the continuation values in $V(h, o, \chi)$ for all origins $o \in \{D, P, N\}$.

3. It affects the aggregate state tomorrow, χ'.

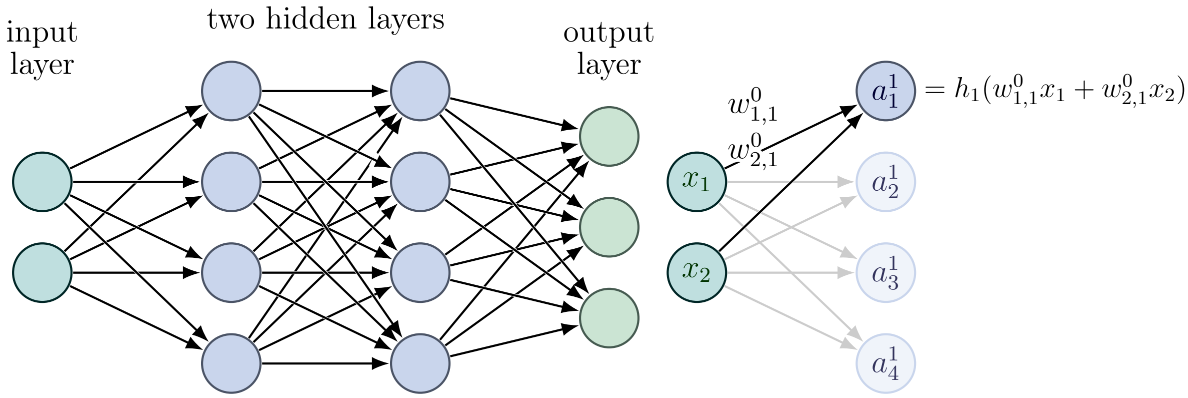
Notice that 2. and 3. constitute a fixed point problem. To compute χ', you need to know the full set of implied choice probabilities. And to compute the choice probabilities, you need to know χ'. This fixed point must be solved for every menu of wages considered when searching for the optimal firm policies. Hence the presence of such a fixed point is particularly computationally burdensome because it would be at the center of many standard solution methods.

I am able to avoid this issue by parameterizing the laws of motion for the state tomorrow χ' given today's state χ and a choice of policies w_D using a neural network. The dominant firm takes this mapping as given when choosing wages. I discuss this in more detail below.

C.2 Neural Networks

In this section, I provide an overview of the basics of neural networks for prediction. For excellent textbook treatments, see Bishop (2006) and Goodfellow, Bengio, and Courville (2016).

Definition. A neural network is a nonlinear function $f : \mathbb{R}^m \to \mathbb{R}^n$ that consists of interconnected nodes, or *neurons*, organized into *layers*. There are three types of layers: the input layer, any number of hidden layers, and the output layer. The number of hidden layers is referred to as the depth of the neural network. Deep learning is commonly used to describe neural networks with more than one hidden layer. The number of nodes is referred to as the width of a layer. In Figure C.1a, I provide an example of a neural network with two inputs, two hidden layers with 4 neurons each, and 3 outputs. This is an example of a *dense* neural network because each node is fully connected to every node in the previous layer.



(a) Network diagram (b) Neuron

Note: Tikz code adapted from `https://tikz.net/neural_networks/`.

Figure C.1: Example of a Neural Network

The simplest neural network is called a perceptron and has no hidden layers. Each output $k \in \{1, 2, \ldots, n\}$ is a linear combination of the inputs,

$$y_k(x, w) = \sum_{i=1}^{m} w^0_{i,k} x_i.$$

The 0 superscript indicates that the weights correspond to the input layer. This neural network has $m \times n$ weights $w^0_{i,j}$ to estimate. However, the universal approximation theorem (Hornik et al. 1989) does not apply to perceptrons. To improve the approximation, we add a hidden layer with $p \in \mathbb{N}$ nodes. This entails applying a nonlinear transformation of the linear combination of inputs using an *activation* function h. The resulting transformations are in turn weighted so that the outputs are now given by

$$y_k(x,w) = \sum_{j=1}^{p} w_{j,k}^1 \underbrace{h\bigg(\sum_{i=1}^{m} w_{i,j}^0 x_i\bigg)}_{a_j},$$

where a_j is the jth neuron of the hidden layer. The universal approximation theorem now applies, but at a cost: there are additional parameters to estimate. We have $m \times p + p \times n$ weights. However, the number of weights scales *linearly* with the number of inputs, as opposed to exponentially with common series such as Chebyshev of Hermite polynomials. This is the reason why neural networks are able to handle high-dimensional functions and bypass the curse of dimensionality. More layers (and/or nodes) can easily be added. For example, the output of the neural network represented in Figure C.1 is given by

$$y_k(x,w) = \sum_{j_2=1}^{4} w_{j_2,k}^2 \overbrace{h_2\bigg(\sum_{j_1=1}^{4} w_{j_1,j_2}^1 \underbrace{h_1\bigg(\sum_{i=1}^{2} w_{i,j_1}^0 x_i\bigg)}_{a_{j_1}^1}\bigg)}^{a_{j_2}^2}, \ k = 1,2,3,$$

where h_1 and h_2 are the activation functions chosen for the first and second hidden layers, respectively. It has $m \times 4 + 4 \times 4 + 4 \times n$ weights.

In addition to the network's depth and width, an important choice is the activation functions. Each layer can have a different activation function. For the hidden layers, the most commonly used are the rectified linear unit (`ReLU`) and the hyperbolic tangent (`tanh`). For the output layer, the choice often depends on the function being approximated. In many economic settings, they can be used to impose constraints. For example, I use a `softplus` activation function for the dominant firm's choice of wages to ensure they are non-negative.

Training. The neural network weights are updated with

$$w^* = \arg\min_{w} \mathcal{L}(x;w),$$

where $\mathcal{L}$ is the *loss* function. The loss function is chosen by the researcher and usually measures the distance between the actual data and model predictions. For example, a commonly used loss function is the mean squared error (MSE)

$$\mathcal{L}^{MSE}(x; w) = \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2,$$

where N is the *batch* size, x_i is the input of the ith data point, $\hat{y}_i = NN(x_i; w)$ is the predicted value, and y_i is the truth. In practice, the weights are updated using gradient descent,

$$w_{new} = w + \eta \frac{\partial \mathcal{L}(x; w)}{\partial w}$$

where $\eta \in \mathbb{R}_+$ is the *learning rate.* η is a key hyperparameter which must be carefully chosen: a high value can update the network too quickly and overshoot the true global minimum, while a small value might not update the network fast enough and get stuck in a local minimum or flat region. Convergence is achieved when the gradient is 0, meaning that there is no change in the weights that would result in a lower loss.

To assess the performance of a neural network, it is common to plot both the *training* and *validation* losses. The training loss is the loss calculated on the training set, a randomly generated dataset used to update the model parameters. The validation loss is the loss computed on the validation set, a randomly generated dataset (different than the training set) that is not used to train the neural network. Usually, the validation loss is computed after the optimization step. Hence, the training loss measures how well the neural network fits the training data and the validation loss captures how well the neural network is performing more generally, i.e. on points that were not used for training. A validation loss greater than the training loss is usually a sign that the model is not able to generalize to points outside of the training set (overfitting). A standard rule of thumb is to tune the neural network architecture and hyperparameters until both losses are roughly equal.

Properties. Neural networks have become widely used due to their attractive properties. Networks with at least one hidden layer are universal function approximators (Hornik et al. 1989), meaning they are able to approximate any continuous function to arbitrary accuracy. Because of their inherent non-linear structure, they can represent highly complex functions with kinks and ridges, binding constraints, non-differentiabilities, discontinuities, and discrete choices. As previously discussed, they can also circumvent the curse of dimensionality because the number of weights to estimate scales linearly with the dimensionality of the input. Furthermore, recent advances in computing make training fast.

C.3 Deep Reinforcement Learning

In this section, I describe the methods from reinforcement learning I use to solve the dominant firm's problem and the planner's problem. More precisely, I use a modified version of the Deep Deterministic Policy Gradient Algorithm of Lillicrap et al. (2015) and Silver et al. (2014). Before describing this procedure, I first provide a brief overview of reinforcement learning and actor-critic algorithms. For a more in-depth textbook treatment of these topics, see Barto and Sutton (2018) and Szepesvari (2010).

C.3.1 Reinforcement Learning

Background. There are three broad paradigms in machine learning: supervised learning, unsupervised learning, and reinforcement learning. Supervised learning is used for labeled data with a clear mapping to predefined outputs such as image classification or spam e-mail detection. Unsupervised learning is employed for discovering patterns or structures in unlabeled data. A widely used example is principal component analysis (PCA), a popular method for dimensionality reduction. Finally, reinforcement learning is used to study an agent who learns to make sequential decisions in an environment to optimize a reward. Deep reinforcement learning refers to the use of deep neural networks in reinforcement learning algorithms.

Setting. For ease of exposition, I borrow notation from the machine learning literature. The canonical setting in reinforcement learning is a Markov Decision Process comprised of a state space $\mathcal{S}$, an action space $\mathcal{A}$, a reward function $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$, an initial state distribution, and Markov transition dynamics for the state. In each period, an agent in state $s_t \in \mathcal{S}$ chooses an action $a_t \in \mathcal{A}$. The agent's behavior is summarized by a policy $\pi : \mathcal{S} \to \mathcal{A}$ which can either be deterministic, $\pi(s) = a$, or stochastic, $\pi(a|s) = P_\pi(A = a, S = s)$. In what follows, I restrict attention to deterministic actions because those are what the dominant firm and the planner choose. The reward obtained from this choice is denoted by $r(s_t, a_t)$ and the agent's discount factor is $\beta \in (0, 1]$. The agent maximizes the present discounted value of returns, $R_t := \sum_{k=0}^{\infty} \beta^k r(s_{t+k}, a_{t+k})$. Two different values are defined:

1. the *state* value $V_\pi(s) = \mathbb{E}[R_t | s_t = s]$ is the expected return from following π, and

2. the *state-action* value $Q_\pi(s, a) = \mathbb{E}[R_t | s_t = s, a]$ is the expected return from choosing a in state s and then following π.

Both formulations are equivalent,[48] but the state-action value is more convenient in practice for the reasons I discuss below. These values can be expressed as standard Bellman equations, e.g. the recursive formulation of the state-action value is

$$Q_\pi(s,a) = r(s,a) + \beta \mathbb{E}_{s'|s,a}[Q_\pi(s', \pi(s'))].$$

The goal of reinforcement learning is to find the policy that maximizes total returns. Notice that many problems studied in economics and related fields fit this structure. I do not provide an exhaustive taxonomy of the growing list of methods developed to solve these types of problems. Instead, I briefly discuss two popular branches in the model-free approach to reinforcement learning, Q-learning and policy optimization, on which actor-critic algorithms build.

Q-Learning. Q-learning[49] is a model-free approach closely related to standard computational methods in economics that solve dynamic programs using the value function. In most applications of reinforcement learning, value function iteration is infeasible because of the dimensionality of the state space. The solution proposed by Q-learning is to estimate the optimal state-action value $Q^*(s,a)$ using a function approximator, $Q(s,a;\theta) \approx Q^*(s,a)$, where θ is the vector of parameters. For example, deep Q-learning refers to the use of a (deep) neural network to do this. There are many variants of this algorithm, but in general, the parameters θ are updated to minimize the Bellman residuals. Importantly, Q-learning is an *off-policy* algorithm, meaning that the actions used to evaluate the fit are not always optimal. A standard approach is to use an $\epsilon-$greedy policy: with probability ϵ, it experiments and randomly selects an action $a \in \mathcal{A}$, and with probability $1-\epsilon$, it selects the optimal action. This *exploration* is key and allows the algorithm to consider points far from the optimal policy that may yield higher returns. ϵ is commonly referred to as the exploitation-exploration parameter with a higher value implying more exploration of the policy space far from the (current) optimal action. In many applications, it is useful to implement a decreasing schedule for ϵ to facilitate convergence. Most Q-learning algorithms also use an *experience replay*, a large buffer that stores previously explored experiences (state, action, reward, and future state) to avoid correlated training samples and improve convergence.

[48] In the case of stochastic actions, the relationship between the two values is

$$V_\pi(s) = \sum_{a\in\mathcal{A}} \pi(a|s) Q_\pi(s,a).$$

For deterministic actions, we have $V_\pi(s) = Q_\pi(s, \pi(s))$.

[49] The name is short for "learning the Q-value".

Policy Optimization. As opposed to value-based methods like Q-learning, policy gradient methods parameterize the policy function. The parameters are updated using the policy gradient theorem (Sutton et al. 1999) which provides an explicit expression for the gradient of the parameterized policy that can be easily computed. When parameterized by a neural network, this is equivalent to setting the loss function equal to the *negative* value implied by the actions.

Actor-Critic Algorithms. Actor-critic algorithms are widely used architectures that lie at the intersection of the value-based and policy-based approaches. Instead of focusing on one or the other, both the policy (actor) and the state-action value (critic) are parameterized and trained jointly. The actor selects actions by exploration and the critic evaluates these actions by computing their expected future rewards, providing feedback to the actor. The actor parameters are typically updated using gradient ascent in the direction suggested by the critic thus increasing the likelihood of actions that lead to higher returns. The critic parameters are updated to minimize a loss capturing the difference between estimated values and actual returns. This combination of value estimation and policy optimization allows agents to learn both what actions to take and how to improve their policy over time, making actor-critic algorithms particularly effective for complex environments.

C.3.2 Deep Deterministic Policy Gradient

In many problems in reinforcement learning, the policy is *stochastic*, i.e. $\pi(a|s) = P_\pi(A = a, S = s)$ is a probability distribution over the set of possible actions. However, in most economic applications including the one in this paper, the policy is *deterministic*: agents choose a single action $\pi(s) = a$. The deterministic policy gradient theorem (Silver et al. 2014) provides an expression for the gradient when the action is deterministic and is a special case of the more general (stochastic) policy gradient theorem discussed above. Deep Deterministic Policy Gradient (henceforth DDPG) is an actor-critic learning algorithm proposed by Lillicrap et al. (2015) that can be used to solve deterministic problems with both a continuous state space and a continuous action space. Both the state-action value $Q(s, a; \theta_Q)$ and the policy $\pi(s; \theta_\pi)$ are approximated with deep neural networks. The critic is updated by minimizing the Bellman residuals and the actor is updated by minimizing the negative value implied by the policy. Random noise is added to the policy for exploration,

$$a = \pi(s; \theta^\pi) + n,$$

where n is typically normally distributed. Moreover, the authors suggest using soft target updates to avoid instability in convergence. Copies Q' and π' of both neural networks are used to compute target values and to update the neural network parameters:

$$\theta_i' = \tau_i \theta_i + (1 - \tau_i)\theta_i', \quad i \in \{Q, \pi\},$$

where $\tau_i << 1$ is the weight placed on the new parameters.

C.4 Solution Method

I separately approximate the following four model objects with their own neural network:

1. NNp_D approximates the dominant firm's optimal choice of wages.
2. NNQ_D approximates the dominant firm's *state-action* value.
3. NNV approximates the worker value.
4. $NN\chi'$ approximates the law of motion for χ'.

Note that the worker value is standard so grid-based methods commonly used in the dynamic discrete choice literature could also be used. However, because of the high dimensionality of the state space, interpolation with grids is both inaccurate (since using an appropriate amount of grid points is infeasible) and slow.

Table C.1: Neural Networks for Decentralized Equilibrium

Description	Mapping	Loss	Depth/Width	Activation	Learning Rate
Firm Policy	$\chi \to w_D$	negative value	3 hidden layers tanh, 128	`softplus`	1e-3
Firm Value	$(\chi, w_D) \to Q_D$	MSE	2 hidden layers leakyrelu, 128	`linear`	1e-3
Worker Value	$(h, o, \chi) \to V$	MSE	3 hidden layers leakyrelu, 128	`linear`	1e-3
Law of Motion	$(\chi, w_D) \to \chi'$	MSE	2 hidden layers leakyrelu, 128	`softmax`	1e-3

Note: All neural networks use the standard ADAM optimizer. MSE (mean squared error) is $\frac{1}{N}\sum_i(\hat{y}_i - y_i)^2$ where $\hat{y}_i$ is the predicted value, y_i is the truth, and N is the number of points in the training data. The `linear` activation function is the identity $f(x) = x$. The `softplus` activation function is $f(x) = \ln(1 + e^x)$. The `softmax` activation function is $f : \mathbb{R}^k \to (0,1)^k$ defined by $f(\mathbf{x})_i = e^{x_i} / \sum_{j=1}^k e^{x_j}$.

Table C.1 reports the neural network configurations. I choose deep (2-3 hidden layers) and wide (128 nodes) architectures. Moreover, I use random weight initializations (He, Zhang, Ren, and Sun 2015) for each neural network to improve convergence. For the firm's policy, I use the `softplus` activation function to ensure the wages chosen are non-negative.[50] I modify the deep deterministic policy gradient method proposed by Lillicrap et al. (2015) and Silver et al. (2014) to solve the firm's problem. As previously discussed in Appendix C.3.2, this is an actor-critic algorithm that approximates both the *state-action* value and the policy with neural networks. Algorithm 1 details how this is done. For each of the N_T training points, I use a Sobol sequence to sample from the unit simplex Δ^{3H} which I then transform using the method proposed by Blank et al. (2021) in section II.B. This method ensures the points are *well-spaced* in the unit simplex. I then compute the firm profits implied by NNp_D, without needing to solve for the fixed point in χ' discussed in Appendix C.1 since the mapping $(w_D, \chi) \to \chi'$ is given by $NN\chi'$. I use these, along with the other neural networks, to compute the firm's labor supply curve. I then compute the profits using target networks NNQ'_D and NNp'_D. As discussed in Appendix C.3.2, the target networks help avoid instability in convergence. Finally, I set the loss to be the mean negative value over all the points in the training dataset.

Algorithm 1: Training NNp_D

Choose N_T
`// Generate the training data of size` N_T
for j *in* $1{:}N_T$ **do**
    Draw χ^j from a Sobol sequence
    Transform χ^j using section II.B of Blank et al. (2021)
    $w_D \leftarrow NNp_D(\chi^j)$
    $\chi' \leftarrow NN\chi'(\chi^j, w_D)$
    Compute m_D from Equation (12) using (χ^j, χ', w_D) and NNV
    $val^j \leftarrow F_D(m_D) + \sum_h a_D^h - \sum_h m_D^h \times w_D^h + \beta NNQ'_D(\chi', NNp'_D(\chi'))$ `// value`
end
`// Train` NNp_D
Update NNp_D parameters to minimize loss $= -\frac{1}{N_T}\sum_j val^j$

For the neural networks approximating the firm value, I use a `linear` activation function.

[50] In principle, there is nothing restricting firms from posting positive wages. In fact, because of the taste shocks and the human capital dynamics, it may even be profitable to offer negative wages. Nonetheless, I rule this out since negative wages are not observable in the data.

The training, detailed in Algorithm 2, is very similar to the policies. The main difference is the loss function used. As with the policy, I use target networks to facilitate convergence. Moreover, as NNQ_D is also used to compute the continuation value, this is effectively equivalent to doing value function iteration (VFI) with neural networks. For NNV, I use one-hot encoding[51] to handle worker origin o, a categorical variable.[52] Just like with the firm values, this amounts to doing VFI with neural networks. Finally, the neural networks for the laws of motion use a `softmax` activation function to ensure the resulting slices of the aggregate distribution sum to one.

Algorithm 2: Training NNQ_D

Choose N_T

`// Generate the training data of size` N_T

for j *in 1:*N_T **do**

  Draw χ^j from a Sobol sequence

  Transform χ^j using section II.B of Blank et al. (2021)

  $w_D \leftarrow NNp_D(\chi^j)$

  $\chi'_D \leftarrow NN\chi'(\chi^j, w_D)$

  Compute m_D from Equation (12) using (χ^j, χ', w_D) and NNV

  $\hat{y}^j \leftarrow NNQ_D(\chi^j, w_D)$ `// predicted`

  $y^j \leftarrow F_D(m_D) + \sum_h a_D^h - \sum_h m_D^h \times w_D^h + \beta NNQ'_D(\chi', NNp'_D(\chi'))$ `// truth`

end

`// Train` NNQ_D

Update NNQ_D parameters to minimize loss $= \frac{1}{N_T} \sum_j ((\hat{y}^j - y^j)/y^j)^2$

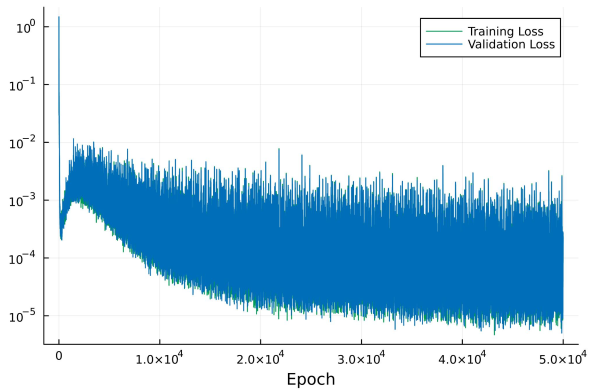
C.4.1 Individual Performance

Before tackling the full equilibrium, I verify that each of the model objects can be properly approximated by the neural networks described above. Note that all these neural networks are deeply interdependent. For example, in Algorithms 1 and 2 it is clear that NNp_D and NNQ_D depend on all the other neural networks. Hence, individually training any of them holds the others fixed.

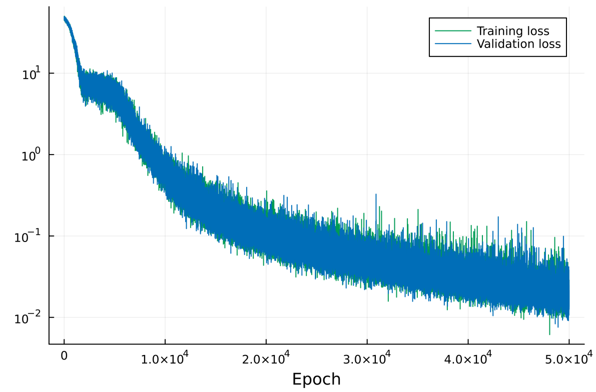
In Figure C.2, I train each of the neural networks individually for 50,000 epochs and plot both the training and validation losses. For all three, the training and validation losses are

[51] This is akin to dummy variables: D is coded as $(1, 0, 0)$, P as $(0, 1, 0)$, and N as $(0, 0, 1)$.

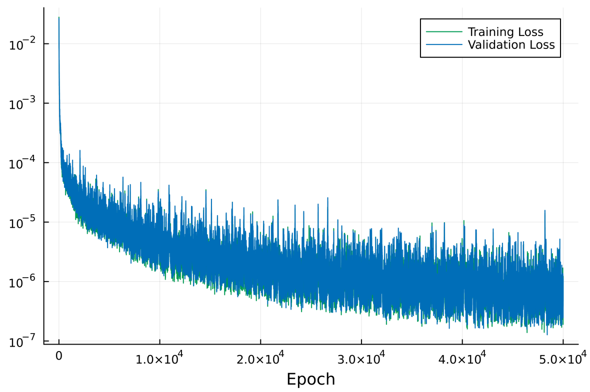
[52] I have also tried using three separate neural networks, each one corresponding to a different origin. I have found this approach to work equally well when training just the worker values, albeit slightly slower. However, when training *all* the neural networks jointly, it is unstable.



(a) NNQ



(b) NNV



(c) $NN\chi'$

Note: The vertical axis is the loss. For each neural network, I plot both the training (green) and validation (blue) losses.

Figure C.2: Independent Training

indistinguishable, suggesting that the neural network architecture and hyperparameters are appropriate. The losses for NNQ quickly decrease and flatten after about 30,000 epochs. In contrast, the losses for NNV and $NN\chi'$ have not stopped decreasing after 50,000 epochs, indicating that the parameters have not yet converged and a better approximation is possible. Nonetheless, for all three, the losses are very small which means the approximations are very precise. For $NN\chi'$, the fit is particularly impressive as the losses are almost machine epsilon.

C.4.2 Joint Training

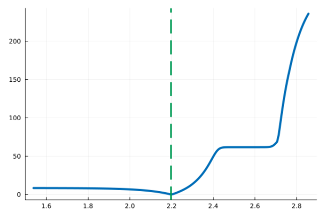
Because of the dependence structure between the neural networks mentioned above, getting them all to converge jointly is tricky. There are two possible approaches. The first is to train all the neural networks jointly until they all converge. The second is to alternate between the firm and worker problems until convergence: hold the firm neural networks constant and train the worker neural networks until convergence, then hold the worker neural networks fixed and train the firm neural networks until convergence. I have found that what works best is training them all jointly. This is not only faster, but also much more stable because it avoids any one of the neural networks training in the wrong direction. For example, if NNV is training holding constant an approximation of the policy NNp_D that is very far from the equilibrium wages, then NNV may converge to something that is also very far from the equilibrium worker value function. Taking that and then feeding it into the training of the firm's policy and value functions may start a vicious cycle from which the algorithm can not recover. Algorithm 3 describes the global solution method used to solve for the equilibrium. I use the sum of all validation losses as the stopping criterion.

Algorithm 3: Solving for the equilibrium

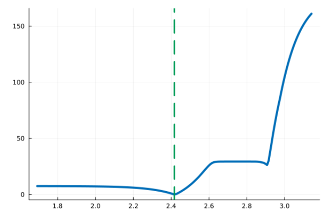
Set the seed and initialize the neural networks
Make copies NNQ'_D, NNp'_D of NNQ'_D, NNp'_D $iter \leftarrow 0$
$val \leftarrow \infty$
while $true$ **do**
  $iter \leftarrow iter + 1$
  `// Train each NN for 1 epoch (see Algorithm 1 and Algorithm 2)`
  Train NNp_D
  Train NNQ_D
  Train NNV
  Train $NN\chi'$
  `// Check for convergence`
  $val \leftarrow$ sum of validation losses
  if $val < tol$ **then**
    break
  end
  `// Update the target networks with weights` τ_Q, τ_p
  $\theta'_Q \leftarrow \tau_Q \theta_Q + (1-\tau)\theta'_Q$
  $\theta'_p \leftarrow \tau_p \theta_p + (1-\tau)\theta'_p$
end

D Estimation Appendix

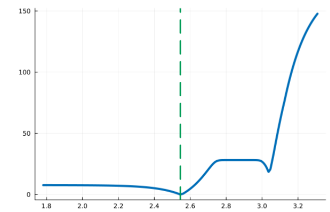
D.1 Slices of objective function



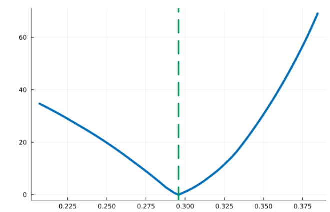
(a) b^1



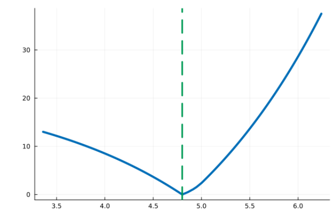
(b) b^2



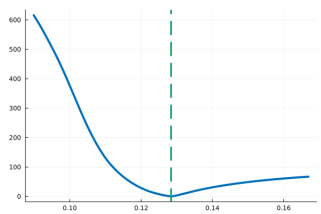
(c) b^3



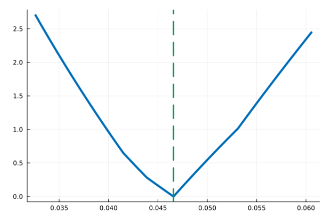
(d) ϕ^{EE}



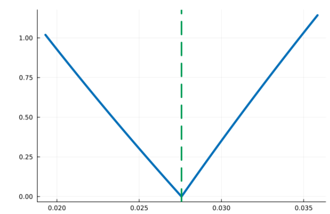
(e) ϕ^{NE}



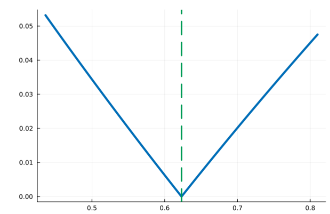
(f) σ



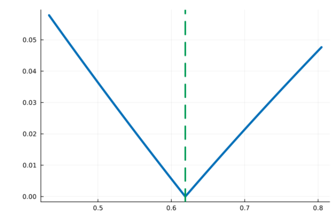
(g) g_E^1



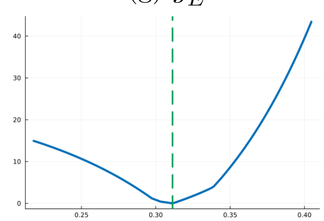
(h) g_E^2



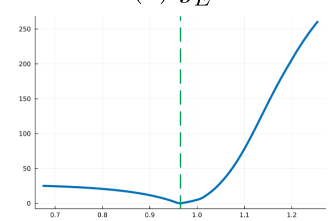
(i) g_N^3



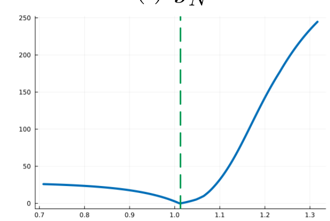
(j) g_N^2



(k) a_D^1



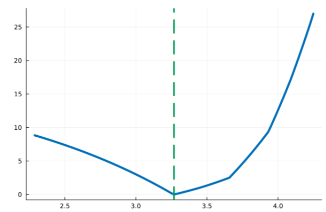
(l) a_D^2



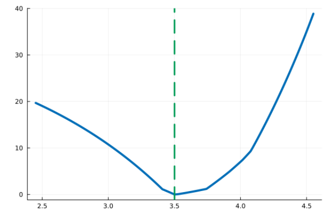
(m) a_D^3

Note: This plots the supply-side estimation objective function around (+/- 30%) the estimated parameter values (dashed green line). The y-axis is in percent deviations.

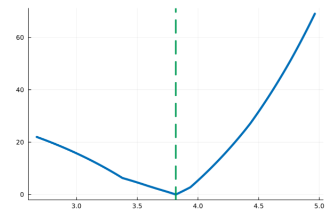
Figure D.1: Slices of the Objective Function for Bottom



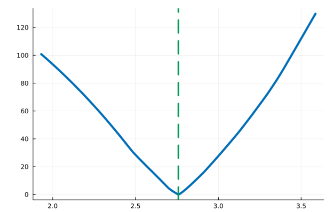
(a) b^1



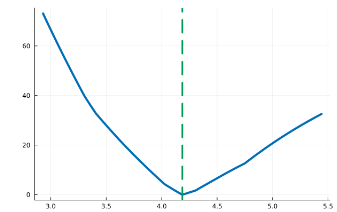
(b) b^2



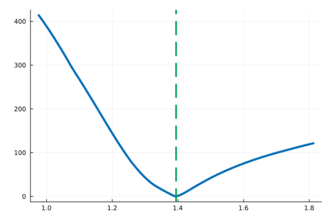
(c) b^3



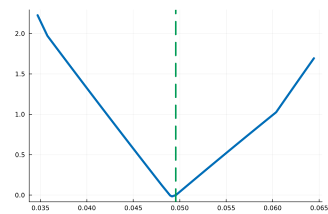
(d) ϕ^{EE}



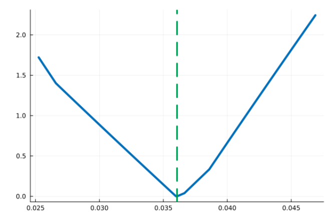
(e) ϕ^{NE}



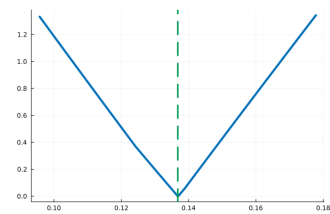
(f) σ



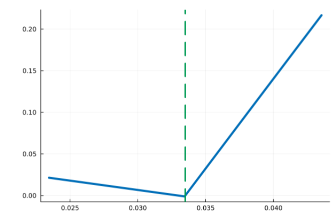
(g) g_E^1



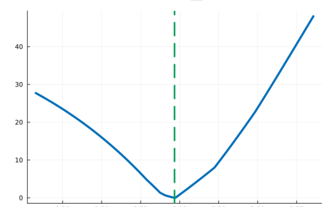
(h) g_E^2



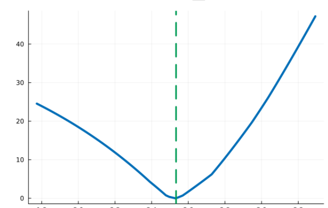
(i) g_N^3



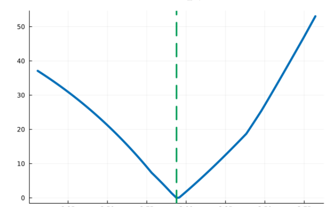
(j) g_N^2



(k) a_D^1



(l) a_D^2



(m) a_D^3

Note: This plots the supply-side estimation objective function around (+/- 30%) the estimated parameter values (dashed green line). The y-axis is in percent deviations.

Figure D.2: Slices of the Objective Function for Top

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